A study on time domain deterministic-stochastic model of spectrum usage in WLAN

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***Abstract*—Duty cycle (DC) has been used to express the deterministic and stochastic aspects of spectrum usage. Specifically, a deterministic model for the mean of the duty cycle (M-DC) has been proposed in a previous work. On the other hand, the observed DC (O-DC) during short time duration has randomness and a stochastic model is used to express the randomness. In this paper, we extend the conventional approach to a combined deterministic-stochastic (DS) model which represents both the deterministic and stochastic aspects. For the distribution of the O-DC, the beta distribution has been used as stochastic model, but we employ a mixture of beta distributions. The mixture-beta distribution can achieve higher accuracy but requires more capacity for data storage in spectrum usage measurements since it has a higher number of parameters than the beta distribution. For this issue, we employ regression analysis in DS-model to reduce the number of parameters while retaining the accuracy. We show the validity of DS-model based on exhaustive spectrum measurements in IEEE 802.11-based wireless local area networks.**

1. Introduction

In the wireless communication field, spectrum scarcity is a pressing issue. For this issue, dynamic spectrum access (DSA) has been investigated since it can utilize the underutilized spectrum resource [1]. In DSA, a secondary user (SU) can utilize spectrum licensed to a primary user (PU), while the spectrum is not occupied by the PU. In DSA there are mainly two key techniques: spectrum sensing to find vacant spectrum and spectrum management techniques, such as spectrum allocation and channel access, to utilize the vacant spectrum efficiently. The key techniques can be designed properly and enhanced by information of PU spectrum usage since this information indicates trends and aspects of the PU spectrum usage. For example, the knowledge of the duty cycle (DC) can enhance spectrum sensing performance [2] and spectrum management [3], [4], [5], [6].

There have been many investigations of spectrum usage modeling in time, frequency and space domains based on measurement campaigns [6], [7], [8], [9], [10]. In these investigations, not only the spectrum utilization ratio, but also the trends and variations of feature quantities in terms of spectrum utilization, such as DC [7], signal strength [11] and busy time (vacant time) [12], [13], have been considered. In this paper, we focus on time-domain spectrum usage modeling in terms of DC.

In spectrum usage measurements with short time duration, such as one second, the observed duty cycle (O-DC) is random. The randomness of O-DC can be expressed by probability density function (PDF), which corresponds to a stochastic model [7]. On the other hand, statistics of DC in the time domain vary over time and this time-varying aspect depends on the spectrum usage (i.e., social behavior and common habits). In [7], it has been shown that the time-varying aspect involves a deterministic behavior. Specifically, the traffic load is typically high in daytime and low in nighttime and the mean of DC (M-DC) is described by a deterministic model [7]. In previous works, the models characterize either the deterministic or stochastic behavior.

In this paper, we extend the previous model in [7] to a deterministic-stochastic-DC (DS-DC) model in which both deterministic and stochastic behaviors can be expressed at once. We propose a spectrum usage model which can describe not only the stochastic behavior, but also the deterministic behavior in the time domain. Typically there is trade-off between accuracy of model and the number of parameters. An efficient model should reproduce the actual distribution accurately while the number of parameters is relatively low. Our main contributions by considering the trade-off are summarized as follows [14]:

- For the PDF of O-DC, the beta and Kumaraswamy distributions have been used as stochastic models [7]. To achieve better accuracy, a mixture-beta distribution, in which two beta distributions are used, is employed as a stochastic DC model.
- Two types of DS-DC models are presented in this paper. In the first DS-DC model, a deterministic model is used for each parameter of the stochastic DC model (i.e., the mixture-beta distribution used for the PDF of O-DC). While the mixture-beta distribution can be more accurate than the beta distribution, the mixture-beta distribution needs more parameters. For this issue, we use polynomial regression analysis to reduce the number of parameters, which is denoted by RDS-DC model and this is our proposed DS-DC model.

2. Measurement setup and methodology

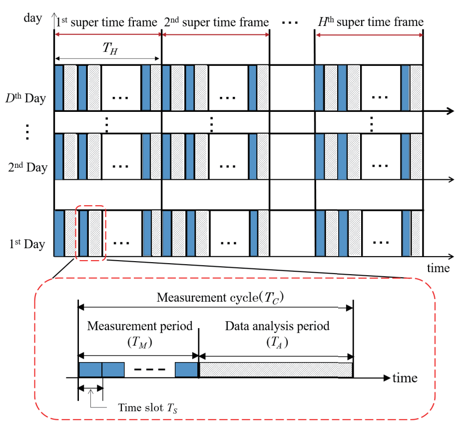


Figure 1: Measurement time schedule

We performed a spectrum usage measurement campaign in a frequency band: W_1 is 2452 - 2472 MHz, mainly utilized by IEEE 802.11 WLAN. The measurement system is located in our laboratory on fourth floor of a building in Koganei-campus, Tokyo University of Agriculture and Technology, Tokyo, Japan (35°41'55.8"N 139°31'00.6"E). The measurement system consists of antennas that can observe the target frequency bands W_1, cables, a real-time spectrum analyzer (RSA) (Tektronix RSA6100A), a network hard disk, a measurement system control computer, and a data analysis computer. The measurement system control computer takes care of the measurement time scheduling, which is shown in Fig. 1. The number of days for spectrum measurement is denoted by D. Since we focus on the daily deterministic behavior of the spectrum usage, the time duration of the deterministic model is set to one day [7]. While deterministic behaviors of spectrum usage in weekdays and weekends may be different, we only focus, without loss of generality, on the spectrum usage during weekdays. We set $D = 29$ days. One day (24 hours) is divided into H super time frames, each of which consists of M measurement cycles. The time durations for one super time frame, and one measurement cycle are denoted by T_H and T_C, respectively. We set T_H and T_C to one hour and one minute, respectively.

One measurement cycle consists of a measurement period and a data analysis period, whose time durations are denoted by T_M and T_A, respectively. Typically T_M depends on the capability of the spectrum measurement devices, such as internal buffer size. During one measurement period, the RSA observes the target frequency W_1 for T_M seconds ($T_M = 100$ ms). Note that the observation of the RSA is continuous unlike typical swept spectrum analyzers. T_M has to be much longer than one continuous spectrum usage cycle, such as data packet, for proper DC estimation in the target frequency band. The time duration for data packet in WLAN is at most about 0.87 ms (corresponding to the time duration of the IEEE 802.11 PLCP (Physical Layer Convergence Procedure) protocol data unit).

The observed data is first stored in the network hard disk and then transferred to the data analysis computer. The data analysis computer provides estimates of the DC by means of fast Fourier transform (FFT)-based energy detection and post processing to achieve accurate spectrum usage detection performance [15]. The estimated DC is denoted by $\Psi_E(c, h, d)$, where c, h, d, indicate the index numbers for the measurement cycle, super time frame, and day, respectively. This $\Psi_E(c, h, d)$ corresponds to O-DC.

The measurement period is divided into N_T time slots and Welch FFT based power spectrum estimation is performed in each time slot. There are N_F frequency bins in one time slot. The time duration for one time slot is denoted by T_S and this time slot corresponds to one Welch FFT time duration. T_S has to be shorter than one continuous spectrum usage cycle (e.g., one data packet). The spectrum usage detections are performed at the data analysis computer based on the estimated power spectrum for $N_T \times N_F$, energy detection (ED), and signal area estimation with false alarm cancellation [15]. The parameters for the Welch FFT based ED are as follows. In Welch FFT, 1024 data samples are divided into 15 segments while the overlap ratio is set to 0.5 [16]. Therefore, the number of frequency bins are set to 128. We set the threshold based on constant false alarm rate criterion where the target false alarm rate is set to 0.01. In this criterion, we need noise floor information in order to set the threshold and we employ forward consecutive mean excision (FCME) algorithm for noise floor estimation [17], [18]. The spectrum usage detection results are denoted by

$$D_{n_T, n_F} = \begin{cases} 1 & \text{(spectrum is occupied)} \\ 0 & \text{(spectrum is vacant)}, \end{cases} \tag{1}$$

where n_T is the time slot index number and n_F is the frequency bin index number. We define a set of index numbers of frequency bin, n_F, involved in W_i as $\mathbf{W}_i$. Now O-DC in the frequency band W_i can be obtained by

$$\Psi_E(c, h, d) = \frac{1}{N_T} \sum_{n_T} \left(1 - \prod_{n_F \in \mathbf{W}_i} (1 - D_{n_T, n_F}) \right). \tag{2}$$

This equation indicates that if a part of the target frequency band $\mathbf{W}_i$ is occupied, the state of the whole target frequency band is detected as occupied as well. In case of OFDM (Orthogonal Frequency Division Multiplexing) based communication, a few sub-carriers can be used at a particular time. In this case, subset of the channel is physically occupied by PU at that time, but the whole channel is reserved for the PU. Therefore, (2) is a convenient and reasonable way to define O-DC for PU protection.

The estimated M-DC at the hth super time frame, $\Psi_E(c, h, d)$, is obtained by averaging over c and d as

$$\Psi_M(h) = \frac{1}{D \cdot M} \sum_d \sum_c \Psi_E(c, h, d), \tag{3}$$

The deterministic behavior of spectrum usage, which depends on social behavior and common habits, is determined in this work by the time schedule in the laboratory: the

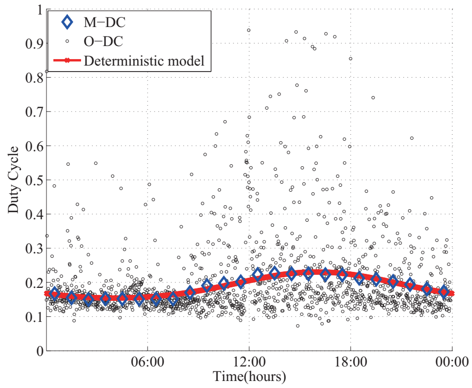


Figure 2: Measurement campaign W_1.

laboratory members arrive in the office at 9:00 and leave anytime between 17:00 and 22:00.

3. Deterministic model for DC

The deterministic model shows the deterministic behavior of M-DC, Ψ_M, and it is defined by [7]

$$F_{\Psi_M}(t) = F_{min} + \sum_{k=0}^{K-1} A_k e^{-(\frac{t-\tau_k}{\sigma_k})^2} \ (0 \leq t \leq T) \qquad (4)$$

where F_{min} is the minimum of $F_{\Psi_M}(t)$, the term $A_k e^{-(\frac{t-\tau_k}{\sigma_k})^2}$ is the kth bell shaped exponential term, A_k is the maximum amplitude for the kth bell shaped exponential term, τ_k is the central time to determine the location of the kth bell shaped exponential term, and σ_k determines the width of the kth bell shaped exponential term. The number of parameters involved in this deterministic model is $N(F_{\Psi_M}|K) = 1 + 3K$. This model corresponds to the deterministic model for low-medium loads proposed in [7].

Now we confirm the validity of the model in (4) based on the measurement campaign. In Fig. 2, $\Psi_M(h)$ (M-DC), $\Psi_E(c, h, d)$ (O-DC), and $F_{\Psi_M}(t)$ is plotted. Each point corresponding to O-DC is obtained by averaging the measurements over one measurement cycle with duration $T_C = 1$ minute. Each point corresponding to M-DC is obtained by averaging the measurements over one super time frame with a duration of $T_H = 1$ hour. In $F_{\Psi_M}(t)$, the least square error criterion is used for parameter fitting. The deterministic model $F_{\Psi_M}(t)$ is fitted to the M-DC points $\Psi_M(h)$. In WLAN, the minimum $\Psi_E(c, h, d)$ is around 0.1, which is caused by periodic beacon signals from WLAN access points (APs).

In the measurement results, the DC is relatively high during the day time as a result of human presence in the laboratory. In addition, in the case of relatively high M-DC, $\Psi_E(c, h, d)$ can have a significantly greater variance and the maximum $\Psi_E(c, h, d)$ can be larger than 0.9. This suggests that the deterministic model may be applicable for the variance of $\Psi_E(c, h, d)$.

4. Stochastic models for DC

4.1. Stochastic model based on Beta distribution for O-DC

In stochastic DC models, typically a basic PDF is employed. The beta distribution is one of the strong candidates to describe the stochastic aspect of O-DC $\Psi_E(c, h, d)$ in the hth super frame [7]. Based on the beta distribution, a stochastic model for O-DC $\Psi_E(c, h, d)$ is given by

$$f_{\Psi_E,B}(x) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1}, \qquad (5)$$

where $0 \leq x \leq 1$, $\alpha > 0$ and $\beta > 0$ are shape parameters, and $B(\alpha, \beta)$ is the Beta function defined by:

$$B(\alpha, \beta) = \int_0^1 z^{\alpha-1}(1-z)^{\beta-1} dz. \qquad (6)$$

The mean and variance of the beta distribution, μ_B and σ_B^2 are given by

$$\mu_B = \frac{\alpha}{\alpha+\beta} \qquad (7)$$

$$\sigma_B^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}, \qquad (8)$$

respectively. The mean, μ_B, from the hth super frame, is equivalent to $\Psi_M(h)$. We employ the Metropolis-Hastings curve-fitting algorithm, which is a Markov chain Monte Carlo method, to set the model parameters [19].

4.2. Stochastic model based on mixture Beta distribution for O-DC

As an alternative model, we propose a mixture-beta distribution in which two beta distributions are used. The stochastic model based on the mixture-beta distribution is defined by

$$\begin{aligned} f_{\Psi_E,B_m}(x) &= \sum_{b=0}^{1} w_b f_{\Psi_E,B_b}(x) \\ &= \sum_{b=0}^{1} w_b \frac{1}{B(\alpha_b, \beta_b)} x^{\alpha_b-1}(1-x)^{\beta_b-1}, \end{aligned} \qquad (9)$$

where w_b is the weighting coefficient for the bth beta distribution ($b \in \{0, 1\}$), $f_{B_b}(x)$ is the bth beta distribution as defined in (5), and $\alpha_b > 0$ and $\beta_b > 0$ are the shape parameters. Since we consider two beta distributions, $w_0 + w_1 = 1$, $w_0 \geq 0$ and $w_1 \geq 0$. The mean and variance for the bth beta distribution are denoted by μ_{B_b} and $\sigma_{B_b}^2$. They are available from (7) and (8), respectively. The parameters are related as follows:

$$\mu_{B_m} = \sum_{b=0}^{1} w_b \mu_{B_b}, \qquad (10)$$

The number of parameters of the beta and mixture-beta distribution are $N(f_{\Psi_E,B}(x)) = 2$ and $N(f_{\Psi_E,B_m}(x)) = 5$, respectively. In this paper, we consider two beta distributions in the mixture-beta model.

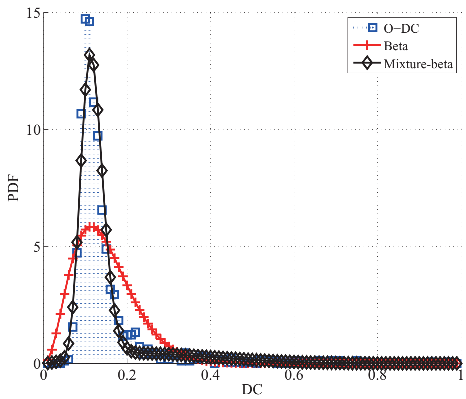


Figure 3: Measurement campaign W_1, Time: 1:00 - 2:00.

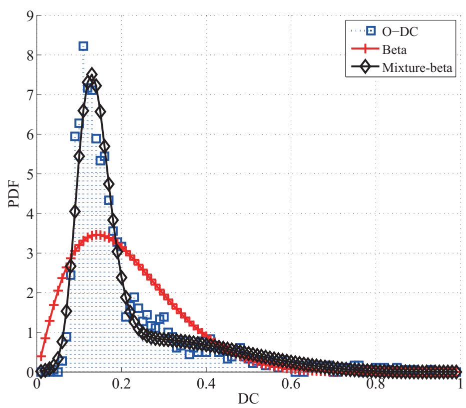


Figure 4: Measurement campaign W_1, Time: 14:00 - 15:00.

4.3. Comparison of stochastic models

We will compare the validity of the stochastic models by means of the Kullback-Leibler divergence (DKL). For discrete probability distributions, $g(x_i)$ and $f(x_i)$, DKL is defined by [7], [20]

$$\mathrm{DKL} = \sum_i g(x_i) \log \left(\frac{g(x_i)}{f(x_i)} \right). \tag{11}$$

This is a measure of the non-symmetric difference between two probability distributions. Figs. 3 and 4 show the measured O-DC at W_1 (WLAN) and the fitted distributions based on the beta (5) and the mixture-beta (9) models, respectively, for different time periods (1:00-2:00 and 14:00-15:00, respectively). In both cases, the model based on the mixture-beta distribution agrees better with the measured O-DC compared to the beta distribution.

5. Deterministic-Stochastic model for DC

As confirmed in the previous section, the deterministic model can express the deterministic behavior of one statistic, such as M-DC, and the stochastic model provides whole

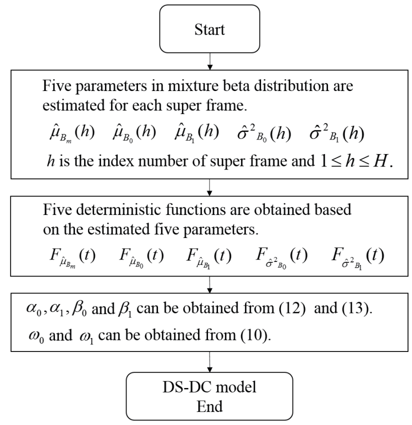


Figure 5: Flow chart for derivation of DS-DC model based distribution parameters.

statistical information at a given time. To express both deterministic and stochastic aspects at once, a DS model for DC is proposed in this section. We will show two DS-DC models based on the mixture-beta distribution. The first model is based on a straightforward approach while the second model is based on regression analysis.

5.1. Deterministic-stochastic model for DC

The DS-DC model is a straightforward approach and the deterministic behavior of each parameter in the stochastic models. In the mixture-beta distribution, five parameters are necessary. We apply the deterministic model (4) to the five parameters. The derivation process for parameters of a distribution based on the DS-DC model is shown below and the flow chart of the derivation is shown in Fig. 5. Specifically, during one super frame, the five parameters can be estimated; such estimates are denoted by $\hat{\mu}_{B_m}(h)$, $\hat{\mu}_{B_0}(h)$, $\hat{\mu}_{B_1}(h)$, $\hat{\sigma}^2_{B_0}(h)$, and $\hat{\sigma}^2_{B_1}(h)$. By curve fitting, such as the least-square method, each parameter in the deterministic model can be specified and the obtained deterministic functions are denoted by $F_{\hat{\mu}_{B_m}}(t)$, $F_{\hat{\mu}_{B_0}}(t)$, $F_{\hat{\mu}_{B_1}}(t)$, $F_{\hat{\sigma}^2_{B_0}}(t)$, and $F_{\hat{\sigma}^2_{B_1}}(t)$. Note that $F_{\hat{\mu}_{B_m}}(t)$ is equivalent to $F_{\Psi_M}(t)$. A set of functions can provide $\mu_{B_m}(t)$, $\mu_{B_0}(t)$, $\mu_{B_1}(t)$, $\sigma^2_{B_0}(t)$, and $\sigma^2_{B_1}(t)$ at time t.

Based on the provided parameters by the set of functions, the PDF of O-DC is available. Therefore, this set of functions constitutes the DS-DC model. ω_0 and ω_1 can be obtained from (10), while α_b can be obtained as

$$\alpha_b = \frac{\mu^2_{B_b} - \mu^3_{B_b} - \mu_{B_b}\sigma^2_{B_0}}{\sigma^2_{B_0}} \tag{12}$$

and β_b can be obtained as

$$\beta_b = \frac{\alpha_b(1 - \mu_{B_b})}{\mu_{B_b}}. \tag{13}$$

In this DS-DC model based on the mixture-beta distribution and the beta distribution, the numbers of parameters are given by $N(f_{\Psi_E,B_m}(x)) \times N(F_{\Psi_M}|K) = 5(1+3K)$ and $N(f_{\Psi_E,B}(x)) \times N(F_{\Psi_M}|K) = 2(1+3K)$, respectively.

5.2. Deterministic-stochastic DC model with regression analysis

To reduced the number of parameters in the model, we propose the RDS-DC model, which employs regression analysis. This idea comes from the fact that statistics in stochastic model may have correlation. Specifically, a larger M-DC leads to a larger variance as confirmed in Fig. 2. In RDS-DC, we use the deterministic M-DC $F_{\hat{\mu}_{B_m}}(t)$. Then, the other parameters, μ_{B_m}, μ_{B_0}, μ_{B_1}, $\sigma^2_{B_0}$, and $\sigma^2_{B_1}$, are obtained by N_Rth order polynomial regression analysis between μ_{B_m} and each parameter, p, as

$$p = \sum_{n=0}^{N_R} B_{n,p} \cdot \mu^n_{B_m}. \qquad (14)$$

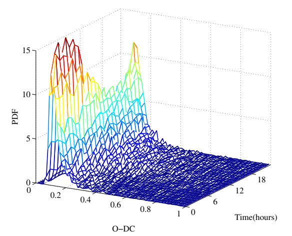
where $B_{n,p}$ is a coefficient for the regression analysis. The coefficients can be obtained by the least-squares method with the measurement results. The estimated parameter p and μ_{B_m} in dth super time frame are denoted by $\hat{\mu}_{B_m}(d)$ and $\hat{p}(d)$, respectively. In this paper, we consider cases of $N_R = 0$ and $N_R = 1$. The number of parameters for RDS-DC is $(N(f_{\Psi_E,B_m}(x)) - 1) \times (N_R + 1) + N(F_{\Psi_M}|K) = 4 \times (N_R + 1) + 1 + 3K$.

5.3. Model verification based on measurement results

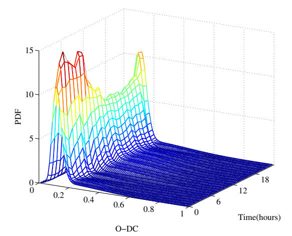
5.3.1. O-DC distributions. To confirm the validity of the DS-DC models, several distributions and models of O-DC are shown in Fig. 6 and the numbers of parameters for each distribution are summarized in Table. 1. In Fig. 6a, the empiric distribution of O-DC in each time (super time frame) is shown. This is a baseline result that the other models attempt to reproduce. In Fig. 6b, the stochastic model with mixture-beta distribution is used to express the distribution of O-DC at each super time frame. This distribution is denoted by Stochastic model based Distribution (SD). SD does not consider the behavior in time domain and corresponds to a conventional approach. We set $K = 2$ for the deterministic models and the mixture beta distribution is used for the stochastic model in Fig. 6a.

The distribution in Fig. 6c is obtained by DS-DC model. This distribution is denoted by DS-DC model based Distribution (DD). In DD, the parameters in the stochastic model are modeled by the deterministic model, i.e., the deterministic model and stochastic model are combined. The results in Figs. 6d and 6e are obtained by RDS-DC models with $N_R = 0$ and $N_R = 1$ in (14), respectively. This distribution is denoted by RDS-DC model based Distribution (RD) with N_R and RD corresponds to our proposed approach.

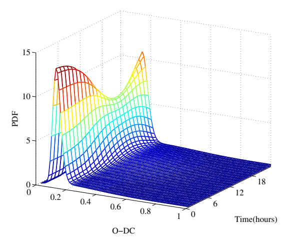
Two points can be confirmed by the results in Fig. 6. First, SD with mixture beta distribution reproduces the empirical distribution (Fig. 6a) adequately, especially the



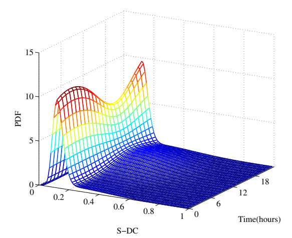
(a) Emprical O-DC distribution at each time (super frame)



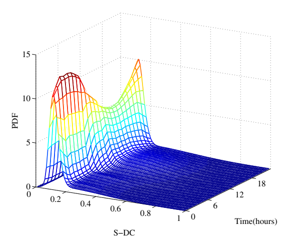
(b) SD with mixture beta distribution



(c) DS-DC model based Distribution (DD).



(d) RDS-DC model based Distribution (RD) with $N_R = 0$.



(e) RDS-DC model based Distribution (RD) with $N_R = 1$.

Figure 6: Distributions of O-DC in W_1

SD can reproduce non-smooth aspects in time domain since it does not have constraints in the time domain. Second, RD with $N_R = 1$ and DD result in similar distributions, however RD with $N_R = 0$ is slightly different. Specifically, the peak of the distribution in time interval from 0:00 to 6:00 in RD with $N_R = 0$ (Fig. 6d) is smaller than the others.

TABLE 1: Number of parameters for each distribution.

Distribution	Number of parameters
SD: Stochastic model based Distribution	120
DD: DS-DC model based Distribution	35
RD: RDS-DC model based Distribution with $N_R = 0$	11
RD: RDS-DC model based Distribution with $N_R = 1$	15

5.3.2. DKL performance. We also evaluate the accuracy of the models numerically by means of DKL between the empirical distribution and the model based distributions. DKL performance as a function of time for W_1 is shown in Fig. 7, where SD with beta distribution is also evaluated.

First, we can confirm that SD with beta distribution achieves the poorest DKL performance. On the other hand, SD with mixture-beta distribution can achieve the best DKL

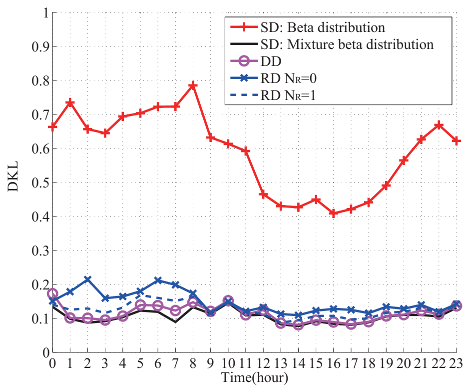


Figure 7: Kullback-Leibler divergences in W_1

performance. This indicates the benefit of the mixture beta-distribution. Second, it can be confirmed that RD with $N_R = 1$ and DD can achieve a similar DKL performance compared to SD with mixture-beta distribution while the number of parameters of RD with $N_R = 1$ is smaller than that of SD. Therefore, it can conclude that RD with $N_R = 1$ is the efficient modeling.

6. Conclusion

In this paper we have investigated models for spectrum usage (DC) in the time domain. In previous works, models for expressing the stochastic and deterministic behaviors of the DC have been investigated separately but not jointly. Specifically, we have proposed several joint deterministic-stochastic models that can express both the deterministic and stochastic behaviors simultaneously. The improved accuracy of the proposed models has been corroborated with empiric data obtained from two long-term spectrum measurement campaigns performed in the WLAN band. Moreover, we have also shown that by means of a regression analysis it is possible to reduce the number of parameters required by the proposed deterministic-stochastic model while preserving a similar level of accuracy.

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