

Experimental Performance Evaluation of Improved Energy Detection under Noise Uncertainty in Low SNR Regime

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Abstract—Improved energy detection (IED) outperforms classical energy detection (CED) as it takes into consideration the statistics of past samples. However, imperfect knowledge of the noise referred to as noise uncertainty (NU) imposes fundamental limitation on the performance of the sensing scheme. NU is prone to degrade the detection performance particularly in the low signal to noise ratio (SNR) regime. In this paper, the analysis of IED under noise power uncertainty in low SNR regime is carried out validated based on the empirical spectrum data of the radio technologies captured using the experimental setup. Obtained results demonstrate that IED outperforms CED in presence of NU, at the cost of nominal increase in the computational time. A gain of 23.2% in the detection performance is observed while using IED over CED at 1 dB NU, 14.13% at 2 dB NU and 16.67% at 3 dB NU.

Index Terms—Cognitive radio, Energy detection, Improved energy detection, Low SNR regime, Noise uncertainty

I. INTRODUCTION

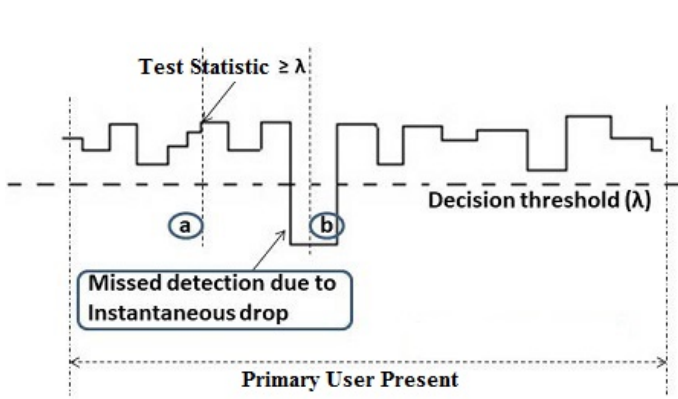
Dynamic Spectrum Access / Cognitive Radio (DSA/CR) is a reliable and effective solution for the improved utilization of the radio spectrum. According to Federal Communications Commission (FCC), most of the available spectrum has been allocated but it is often significantly underutilized. To mitigate the spectrum unavailability issue arising out of the static and exclusive frequency allocation, CR has been looked upon as a potential solution. CR can be defined as a radio network technology that is aware of its operational and geographical environment and adapts to it intelligently [1], [2]. CR allows unlicensed (secondary) users to utilize the spectrum temporarily untapped by the licensed (primary) users in a non-interfering manner.

The ability to reliably and autonomously identify unused frequency bands is envisaged as one of the main functionalities of CRs [3]. There are plentiful techniques for determining the channel state [4]–[8], of which energy detection (ED) is the most popular technique owing to its simplicity and low implementation and computational costs. Moreover, ED does not require any prior knowledge of the primary user signal [3]. Among the algorithms proposed so far for spectrum sensing, improved energy detection (IED) performs better than classical

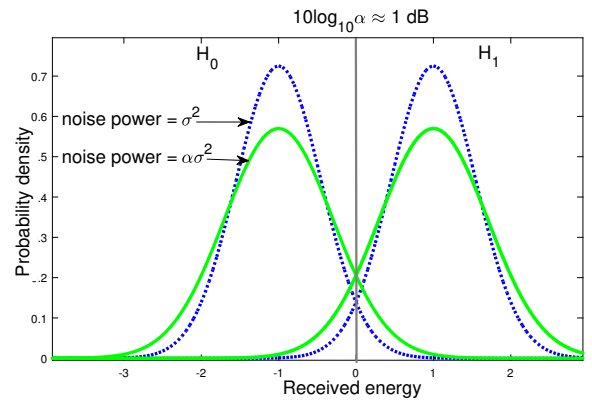
energy detection (CED) and modified energy detection (MED) [9]. The IED preserves a similar level of complexity and computational cost as compared to CED. The interest of this work is in the IED which outperforms the previously proposed algorithms owing to its consideration of average test statistics as well as test statistic of previous event apart from instantaneous test statistic [9].

The spectrum sensing decision depends upon many factors, with the noise uncertainty (NU) being one of them. As uncertain factors commonly exist in the practical networks, perfect knowledge of the noise level is not available. Noise is an aggregation of temperature variation, noise power calibration error, quantization noise, etc. Therefore, it is not likely that noise power would remain constant during the sensing duration. The ED performance degrades heavily under the low SNR and NU conditions [10], [11], which can restrict its efficiency for CR [8]. Indeed there exists an SNR wall below which ED is unreliable [12]. Basically, the performance of the ED algorithm depends greatly on the SNR level of the received signal. The spectrum sensing system with dynamic noise variance can be depicted with the dynamic state-space model [13]. A small error made in the initial stage of spectrum sensing may propagate further and can significantly affect the output. Thus, in practice uncertainties cannot be neglected and must be considered while working upon the spectrum sensing.

Previously, the potential effects of the particular primary signal properties on the resulting detection probability of CED have been analyzed in CR networks [14]. NU being a reason for the performance degradation of the system cannot be ignored and needs to be considered during the practical examination. IED being a realistic spectrum sensing approach with better detection performance even in a low SNR regime, when used to tackle the NU gives better detection performance. The performance of IED in the presence of the NU has not been investigated yet in the literature. In this context, this work provides a detailed mathematical and experimental performance evaluation of the IED algorithm under NU to determine whether this version of the ED algorithm does lead towards an improved signal detection under worst uncertain



(a) Motivation for IED.



(b) Distribution of received energy in presence and absence of NU.

Fig. 1: Rationale for IED and Noise Uncertainty.

scenarios. The major contributions of this work are as follows:

- Firstly, a comprehensive analysis of the fundamental bounds on the detection performance in low SNR regime in the presence of NU while using IED technique for spectrum sensing have been carried out.
- Secondly, the validity and accuracy of the obtained analytical results are corroborated using empirical measurement data obtained with an experimental hardware setup.
- Thirdly, we investigate the computational time cost of both methods (CED and IED) under both scenarios (with and without uncertainties) and demonstrate that the computation complexity is not affected significantly by the presence of uncertainty. Furthermore, the improved performance of IED is obtained at the cost of nominal increase in the computation time.

The rest of the paper is organized as follows. Section II provides an overview of the theoretical performance of ED under NU environment. With a brief discussion of spectrum sensing and ED algorithm, the concept of NU is examined. The consequences of NU are expressed in form of mathematical equations and graphs. Section III provides a detailed view of the experimental set up used for acquiring data of different radio technologies. Further, section IV provides the experimental results for validating the theoretical analysis. Finally, section V draws conclusion of this work.

II. THEORETICAL PERFORMANCE OF IED UNDER NOISE UNCERTAINTY

A. System Model

The decision to be made regarding the occupancy of the channel can be represented with a binary hypothesis H_0 (null hypothesis) and H_1 (alternative hypothesis).

$$\begin{aligned} H_0 : y(n) &= w(n) & n &= 1, 2, 3, \dots, N, \\ H_1 : y(n) &= x(n) + w(n) & n &= 1, 2, 3, \dots, N, \end{aligned} \quad (1)$$

where $y(n)$ represents the received signal at n^{th} instant,

$w(n)$ is the Additive White Gaussian Noise (AWGN) and $x(n)$ represents the transmitted signal. Here H_0 is a null hypothesis stating that there is no primary signal in the sensed spectrum band, and hypothesis H_1 indicates that some licensed user signal $x(n)$ is present. N denotes the number of samples collected during the signal observation interval (i.e., the sensing sample size).

B. Motivation for the proposed analysis

Fig. 1 provides the rationale for using IED and NU in our work. The performance of detection probability is degraded due to false alarms and mis-detections. IED improves the detection performance by avoiding the mis-detections caused by the sudden changes in the signal energy by considering the present event and the average of past L sensing events for calculating the test statistics as shown in Fig. 1(a). Consider the scenario as shown in Fig. 1(a) where there is a sudden drop in energy of signal at an instant 'b' leading to a mis-detection. In IED the misdetections can be avoided upon using the average of the test statistics of past L events as in case of averaging, the test average will be greater than decision threshold resulting in declaring the channel to be busy (i.e, hypothesis H_1).

The probability of the detection (P_d) of IED under AWGN channels as a function of the SNR is given by [9]¹

$$P_d^{IED}(\gamma) = P_d^{CED}(\gamma) + P_d^{CED}(\gamma) \left(1 - P_d^{CED}(\gamma)\right) \xi(\gamma), \quad (2)$$

where Q represents the Gaussian Q - function [15] and

$$P_d^{CED}(\gamma) = Q\left(\frac{Q^{-1}(P_{fa})\sqrt{2N} - N\gamma}{\sqrt{2N}(1 + \gamma)}\right), \quad (3a)$$

$$\xi(\gamma) = \frac{Q^{-1}(P_{fa})\sqrt{2N} - \frac{MN\gamma}{L}}{\sqrt{\frac{2N}{L}(1 + \frac{M}{L}[(1 + \gamma)^2 - 1])}}, \quad (3b)$$

¹The results from [9] were obtained considering real sampling (instead of the more commonly used complex sampling) and as a result of that there are some $2N$ terms that should be N when complex sampling is used.

where P_{fa} denotes the probability of the false alarm, N is the sensing sample size, γ denotes the SNR, L is the total number of last sensing events considered in IED and M represents the number of sensing events where the primary signal was actually present and it varies from 0 to L .

In a low SNR regime ($\gamma \ll 1$) the above expression can be approximated as:

$$P_d^{CED}(\gamma) = Q\left(Q^{-1}(P_{fa}) - \sqrt{\frac{N}{2}}\gamma\right), \quad (3c)$$

$$\xi(\gamma) = Q^{-1}(P_{fa})\sqrt{L} - M\sqrt{\frac{N}{2L}}\gamma. \quad (3d)$$

C. Noise uncertainty model

Practical networks are susceptible to NU, mostly resulting from varying thermal noise in components caused by temperature variations (non-uniform, time-varying), noise due to transmissions by other users or noise power calibration errors. Fluctuation of noise power is considered to be NU [16]. The noise power is uncertain and can be estimated within a range [17] as $\hat{\sigma}_w^2 \in [\sigma_w^2, \alpha\sigma_w^2]$, where $\hat{\sigma}_w^2$ represents the estimated noise power, σ_w^2 represents the nominal noise power and α is the NU parameter, such that $\alpha > 1$ [14]. A detailed explanation of P_d^{CED} has been provided in [14].

As shown in Fig.1(b), in the presence of NU the Gaussian curve flattens, which in turn increases the probability of the error. As $\alpha > 1$, the worst case scenario would be one where the estimated noise power is $\hat{\sigma}_w^2 = \alpha\sigma_w^2$. The ED decision threshold in presence of NU can be expressed as [9]:

$$\lambda = \left(Q^{-1}(P_{fa})\sqrt{2N} + N\right)\alpha\sigma_w^2.$$

The mean and average values taken are,

$$\mu_{avg} = \frac{MN}{L}(\sigma_x^2 + \sigma_w^2) - \frac{(L-M)}{L}N\sigma_w^2,$$

$$\sigma_{avg} = \sqrt{\frac{MN}{L^2}(\sigma_x^2 + \sigma_w^2)^2 + \frac{(L-M)}{L^2}2N\sigma_w^4},$$

where σ_x^2 and σ_w^2 are the signal and noise power respectively. The $P_d(\gamma)$ expression in (3) is of the form $P_d(\gamma) = Q\left(\frac{\lambda - \mu_{avg}}{\sigma_{avg}}\right)$. Upon considering NU while computing the decision threshold, mean and the average values, the expressions in (3a)-(3b) then becomes:

$$P_d^{CED}(\gamma) = Q\left(\frac{\alpha Q^{-1}(P_{fa})\sqrt{2N} - N(\gamma + 1 - \alpha)}{\sqrt{2N}(1 + \gamma)}\right), \quad (4a)$$

$$\xi(\gamma) = \frac{\alpha Q^{-1}(P_{fa})\sqrt{2N} + N\alpha - \frac{MN\gamma}{L} - N}{\sqrt{\frac{2N}{L}\left(1 + \frac{M}{L}\left[(1 + \gamma)^2 - 1\right]\right)}}. \quad (4b)$$

In a low SNR regime ($\gamma \ll 1$)

$$P_d^{CED}(\gamma) \approx Q\left(\alpha Q^{-1}(P_{fa}) - \sqrt{\frac{N}{2}}(\gamma + 1 - \alpha)\right), \quad (4c)$$

$$\xi(\gamma) = \alpha Q^{-1}(P_{fa})\sqrt{L} - M\gamma\sqrt{\frac{N}{2L}} + (\alpha - 1)\sqrt{\frac{NL}{2}}. \quad (4d)$$

In a low SNR regime, $(\gamma + 1) \approx 1$. Upon approximating equations (4a) and (4b) for low SNR values, equations (4c) and (4d) are obtained respectively.

It can be observed from (4) that, as $\alpha \geq 1$ and the Q -function is decreasing in nature, P_d decreases under NU and degrades further with an increase in uncertainty α . With an increase in the sensing sample size N the value of the Q -function increases and so does the detection probability. It also depends on the operating parameters of target P_{fa} , M and L .

III. MEASUREMENT SETUP AND DATA ACQUISITION

The performance of the spectrum sensing algorithm of IED has been verified in experiments carried out by means of USRP board [18]. A measurement setup for the data acquisition was deployed on the roof-top of School of Engineering and Applied Science (SEAS), Ahmedabad University.

The empirical test bed comprises of two measurement setups as shown in Fig. 2. The measurement setup-I consist of Discone antenna (D3000N) and digital spectrum Analyzer (Rigol DSA-875) connected with a Personal Computer (PC) system. This setup is used to analyze the presence or absence of PU, by observing the power spectral density on the spectrum analyzer screen when tuned to a particular band. Setup-I is more useful in case of discontinuous transmitters (GSM-900 in our case). Measurement setup-II consists of Discone antenna (D3000N), USRP (N-210) and a PC system running GNU-Radio software is as shown in Fig. 2. Python script is executed in GNU radio for acquiring the (I/Q) data from the USRP. Once the data are captured, off-line processing is performed in MATLAB and then the proposed scheme is applied to the stored data to check its validity. Table I shows the details of USRP configuration and channels captured in this study while Table II indicates the tuning parameters of the spectrum analyzer.

Furthermore, similar to [19], the impact of fading was minimized to make sure that the transmission power pattern was the superior aspect in the received power variability. The constant transmission power pattern is analogous to the AWGN channel i.e., a channel with no fading when the SNR is constant [14], and this helps in the validation of our proposed analysis onto the empirical spectrum data set.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

This section analyzes the theoretical and empirical performance of the IED method when applied to E-GSM 900 signal. The theoretical relation of the detection probability and the sensing sample size N is verified analytically for different values of N . Here, for the ease of calculations, N is taken to be 10, 100 and 1000. Results are derived considering 1 dB NU. Under the NU conditions, parameter α can take any value greater than or equal to 1.

Fig. 3 shows the theoretical and empirical detection probability using CED and IED method in absence of NU, while, Fig. 4 shows the counterpart in presence of 1-dB NU for different values of M ranging from $[0, L]$ for $L = 3$. As it can be appreciated in Fig. 3 and 4, a significant difference

TABLE I: USRP configuration and channels measured in this study.

Radio Technology	Channel Number	F_{start} (MHz)	F_{center} (MHz)	F_{stop} (MHz)	Signal Bandwidth (MHz)	Gain (dB)	Decimation Rate	Sampled Bandwidth (MHz)
FM Broadcasting	-	96.500	96.700	96.900	0.2	45	64	1
UHF Television Band-IV	U - 33	566	570	574	8	45	8	8
E-GSM(900) DL	77	950.2	950.4	950.6	0.2	45	64	1
DCS (1800) DL	690	1839.6	1840.8	1841	0.2	45	64	1

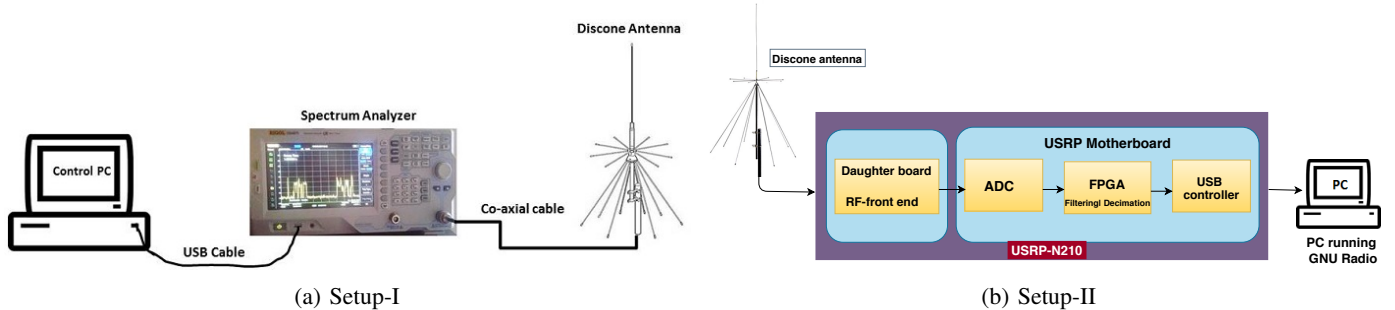


Fig. 2: Empirical test bed setup for spectrum data acquisition.

TABLE II: Parameters of spectrum analyzer

Parameter	Value
Frequency range	75-2000 MHz
Frequency span	45-600 MHz
Frequency bin	Depends on band selected
Resolution Bandwidth-RBW	10 kHz
Video Bandwidth-VBW	10 kHz
Measurement period	5-15 mins
Sweep time	1 second
Scale	10 dB/division
Input attenuation	0 dB
Detection type	RMS detector

in P_d can be observed at low SNR regime. The fundamental limitation on the detection probability caused by the NU is because in a low SNR environment the noise component dominates over the signal component which resembles the hypothesis H_0 and this results in an increase in the detection probability. On the contrary, when the SNR increases, more test statistics will lie below the threshold thereby resulting into a decrease in the detection probability. The gray line in Fig. 4 represents the SNR wall. Below the SNR Wall the primary signal cannot be detected irrespective of the sensing sample size [12]. It can be observed from Fig. 3 that the theoretical plot overlaps exactly with the empirical plot for $M = 2$. While it can be noticed that in Fig. 4 a slight deviation is seen between the empirical and the theoretical plot for $M = 2$. This deviation is observed because while performing the experiment we cannot have a perfect measure of uncertainty. It is difficult to get exactly 1 dB NU while performing the experiment, thus justifying the minor deviation in the analytical and empirical plots for $M = 2$.

Fig. 5 depicts the receiver operating characteristic curve of IED under 1-dB NU computed theoretically and empirically. The three curves show different probability of detection for

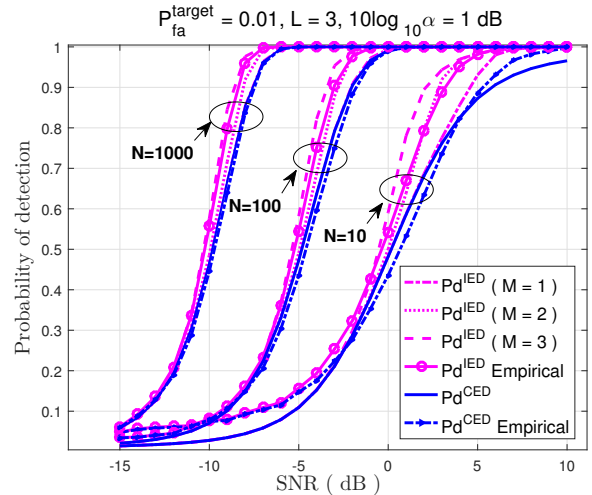


Fig. 3: Theoretical and empirical performance of IED and CED without NU ($M = [0, L]$).

different values of low SNR (-4.3 dB, -5 dB, -5.7 dB). A sample size of 1000 is taken so as to reduce the signal variability and resemble it with the true signal energy. The probability of false alarm is taken in a range of 0.01 to 0.99. When the estimated noise power $\hat{\sigma}_n^2$ is larger than the nominal noise power σ_n^2 , i.e α dB NU, the P_{fa} is likely to increase as more test statistics will lie above the threshold than usual. With an increase in the SNR value, the curve tilts more towards left, i.e detection probability is high as compared to lower SNR values.

Fig. 6 shows a graph of P_f versus P_{md} in IED and CED under 1 dB NU for different values of M ranging from 0 to L . For a given P_{fa} , P_{md} is minimum in IED with $M = 3$ (when $L = 3$) and increases with decrease in value of M . With an

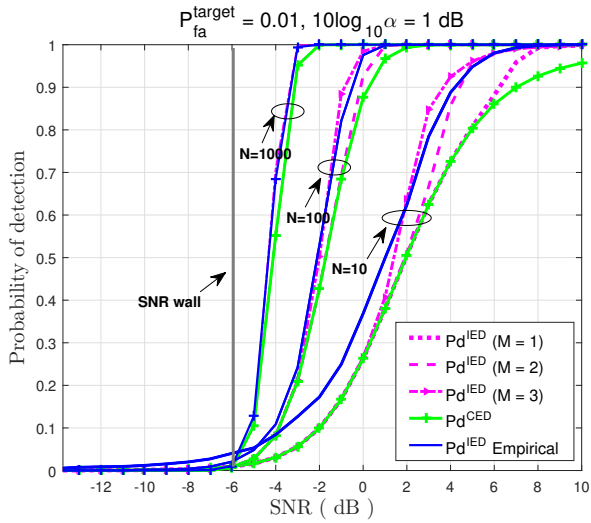


Fig. 4: Theoretical and empirical performance of IED and CED under 1-dB NU ($M = [0, L]$).

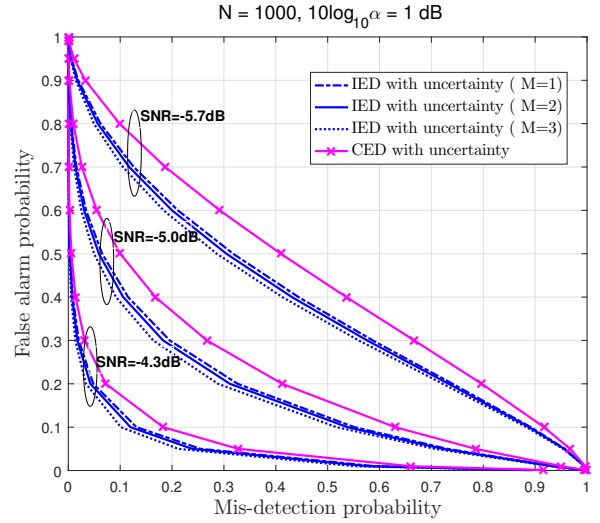


Fig. 6: Theoretical and empirical performance of P_{fa} v/s P_{md} in IED and CED for $M = [0, L]$.

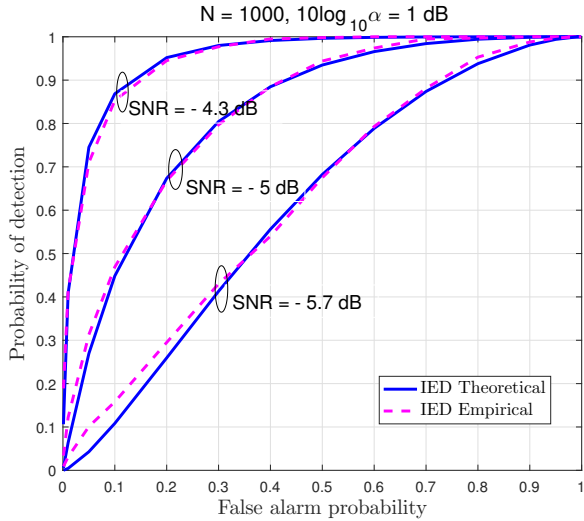


Fig. 5: Theoretical and empirical ROC of IED at 1 dB NU.

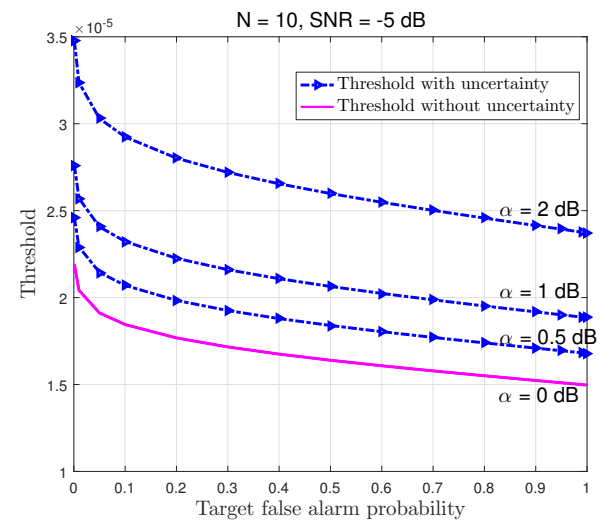


Fig. 7: Threshold in IED and CED at different values of NU.

increase in the sample size, P_d increases thereby reducing the P_{md} value and tilting the plot towards the left. A sample size of 1000 is taken so as to reduce the signal variability and resemble it with the true signal energy.

Fig. 7 shows the behaviour of the threshold for a given P_{fa} at different NU levels at the SNR of -5 dB. In ED, the threshold is usually set using constant false alarm rate approach with an optimum trade off between P_{md} and P_{fa} . However, due to dependency on noise power, it varies from its ideal value depending upon the NU. With an increase in NU, the threshold is raised leading to a reduced P_d . The threshold is proportional to $Q^{-1}(P_{fa})$ and follows an almost linear relation. At high values of P_{fa} , the threshold is set to a lower value so as to increase P_d . The sample size of 10 has been selected to provide a high P_{fa} .

Fig. 8 shows the computational time cost of CED and IED for different sample sizes. The computation time cost of the IED algorithm is higher than that of CED algorithm due to taking into consideration the previous instances. The average computation time for these algorithms is calculated in MATLAB for different sample sizes. Monte Carlo loop for 8000 iterations have been carried out with $P_{fa} = 0.01$, $L = 3$ and 1 dB NU. The experiment is carried out in Intel i5 Quad core processor at 2.66 GHz. From Fig. 8, it can be observed that IED in presence of NU does not add any further computational cost.

Fig. 9 depicts the percentage gain in the detection performance of CED and IED at different values of NU in a low SNR Regime. The gain in the detection performance of IED over CED is 23.3% at 1 dB NU, 14.13% at 2 dB NU and 16.67

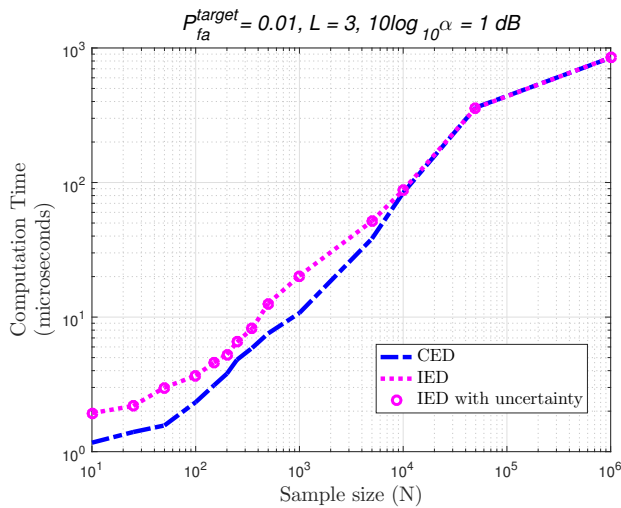


Fig. 8: Computational time complexity in IED and CED for different sample size.

% at 3dB NU in a low SNR regime. The detection probability has been computed at an SNR of 1 dB, P_{fa} of 0.1 and the sample size of 1000.

V. CONCLUSION

In this work, analysis of IED under noise power uncertainty in low SNR regime is carried out and evaluated experimentally based on empirical spectrum data. We conclude that IED outperforms CED in presence of NU. The effect of noise power uncertainty is quantified in the results. Furthermore, a gain of 23.2% in the detection performance is observed while using IED over CED at 1 dB NU, 14.13% at 2 dB NU and 16.67% at 3 dB NU. However, the performance improvement is at the cost of nominal increase in the computational time. NU at the receiver is a realistic consideration, that needs to be considered in the design and development of CR systems.

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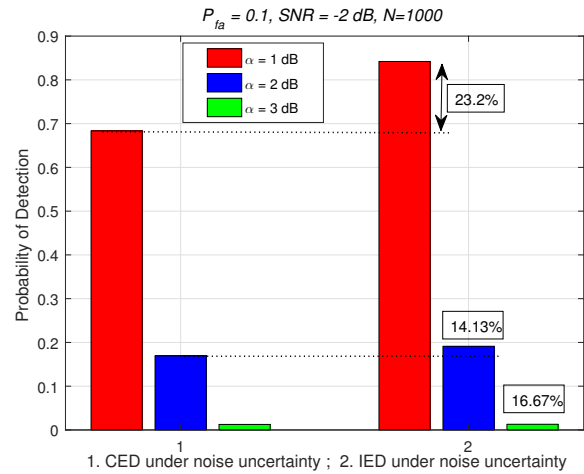


Fig. 9: Improvement in detection performance using IED.

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