# Can Primary Activity Statistics in Cognitive Radio be Estimated under Imperfect Spectrum Sensing?

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***Abstract*—Owing to the opportunistic nature of the Dynamic Spectrum Access (DSA) paradigm based on the Cognitive Radio (CR) concept, the behaviour and performance of DSA/CR systems depend on the spectrum occupancy pattern of the primary system. DSA/CR systems can monitor periodically the occupancy state of licensed channels in order to gain statistical information on their occupancy patterns, and exploit this information in decision-making processes. Based on the outcomes of periodic spectrum sensing decisions, DSA/CR systems can make an estimation of the duration of the channel idle/busy periods and compute relevant statistics such as the minimum period duration, the moments (e.g., mean and variance) and the underlying distribution. However, the imperfect performance of spectrum sensing methods may lead to inaccuracies on the estimated statistics. In this context, this work carries out a detailed simulation-based study on the impact of imperfect spectrum sensing performance on the estimation of the primary activity statistics. The relative impact of various parameters is analysed and quantified, and guidelines to properly configure the spectrum sensing function of DSA/CR systems are provided. Moreover, several methods to overcome sensing errors are proposed and evaluated.**

## I. Introduction

The Dynamic Spectrum Access (DSA) paradigm relying on the Cognitive Radio (CR) concept [1, 2], has the potential to improve the spectrum efficiency by permitting unlicensed (secondary) users to access, in an opportunistic and non-interfering manner, licensed spectrum bands during the inactivity periods of the licensed (primary) users. As a result of the opportunistic nature of this spectrum access principle, the behaviour and performance of DSA/CR systems is tightly dependent on the spectrum occupancy patterns of primary systems. Therefore, the knowledge of the statistical properties of such occupancy patterns represents a valuable information that can be exploited in several ways by DSA/CR systems, including for instance the prediction of future trends in the spectrum occupancy [3, 4], the selection of the most appropriate band/channel of operation [5–9], or the decision on future actions to optimise the system performance and improve the spectrum efficiency [10–14].

The activity statistics of a primary channel are initially unknown to the DSA/CR system but can be estimated based on spectrum sensing decisions. Spectrum sensing methods make a binary decision on the idle/busy state of a channel based on a set of signal samples of the channel. The sequence of observed idle/busy states can be used to make an estimation of the durations of the idle/busy periods. By means of an appropriate processing of the observed period durations, the DSA/CR system can make an accurate estimation of the channel activity statistics such as the minimum period duration, the statistical moments (e.g., mean, variance, etc.) or the underlying distribution. However, the accuracy of the estimated statistics depends, among other factors, on the performance of the employed spectrum sensing method. Unfortunately, existing spectrum sensing methods are imperfect in practice, meaning that some sensing errors may occur occasionally. In particular, idle channels may sometimes be reported as busy (event referred to as false alarm), while busy channels may in some cases be reported as idle (missed detections). Sensing errors alter the sequence of idle/busy states observed in the channel and therefore affect the estimated period durations. As a result, sensing errors can severely affect the estimated channel activity statistics and their accuracy, thus limiting their practical utility. In this context, this work performs, by means of simulations, a detailed analysis on the impact of imperfect sensing performance on the estimation of primary activity statistics in DSA/CR systems. The relative importance of various parameters is analysed and quantified, and guidelines to properly configure the spectrum sensing function of DSA/CR systems are provided. Moreover, several methods to palliate the degrading effects of spectrum sensing errors on the estimated statistics are proposed and assessed as well.

The rest of this work is organised as follows. First, Section II provides a formal description of the problem under study. Section III presents the considered simulation approach. The impact of the imperfect sensing performance on the estimated minimum period duration (Section IV), the estimated moments (Section V), and the estimated distribution (Section VI) is then analysed. Several methods to mitigate the adverse consequences of spectrum sensing errors are proposed and evaluated in Section VII, and a discussion on the configuration of spectrum sensing based on the obtained results is provided in Section VIII. Finally, Section IX concludes the paper.

## II. Problem formulation

The estimation of the duration of the idle/busy periods of a channel based on spectrum sensing decisions is illustrated in Figure 1. DSA/CR users sense the channel with a finite sensing period $T_s$ (shorter than the minimum period duration). In every sensing event, a binary decision on the idle ($\mathcal{H}_0$) or busy ($\mathcal{H}_1$) state of the channel is made. When the observed channel state changes, the time interval elapsed since the last state change is computed as shown in Figure 1(a) to make an estimation $\widehat{T}_i$ of the real period duration $T_i$ ($i = 0$ for idle periods, $i = 1$ for busy periods). The individual period durations (and the resulting statistics) can be estimated within reasonable accuracy levels provided that spectrum sensing decisions $\mathcal{H}_i$ are correct. A Perfect Spectrum Sensing (PSS) scenario without sensing errors can be assumed when the primary Signal-to-Noise Ratio (SNR) at the DSA/CR receiver

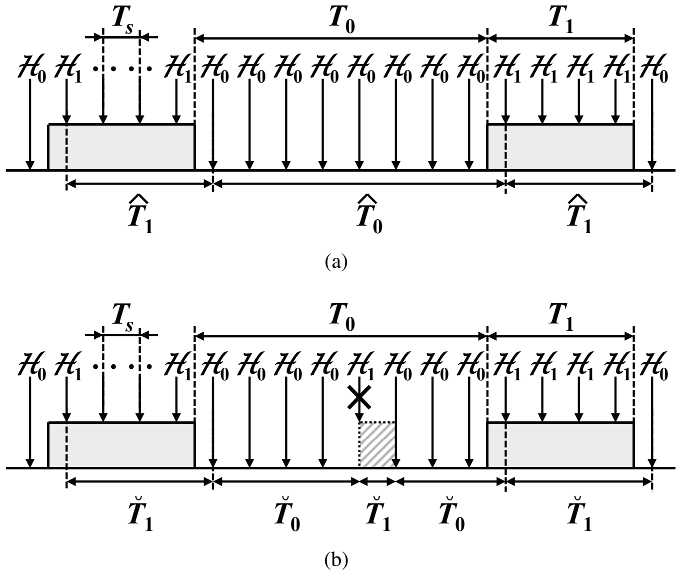


Fig. 1. Estimation of period durations from spectrum sensing decisions: (a) under perfect spectrum sensing, (b) under imperfect spectrum sensing.

is sufficiently high. Under low SNR conditions, however, an Imperfect Spectrum Sensing (ISS) performance results in some occasional sensing errors, which leads to incorrectly estimated period durations $\breve{T}_i$. As shown in Figure 1(b), an idle period may be observed as a sequence of three periods (idle-busy-idle) as a result of a false alarm. Similarly, a missed detection may result in a busy period being reported as a sequence of busy-idle-busy periods. Incorrect period durations $\breve{T}_i$ may lead to inaccurate estimations of the channel activity statistics. The objective of this work is to analyse the impact of sensing errors resulting from an ISS performance on the accuracy of estimated primary activity statistics such as the minimum period duration, the mean and variance, and the distribution.

A number of factors affect the estimated primary activity statistics under ISS. First of all, the estimated period durations $\breve{T}_i$ depend on the number $K$ of sensing errors in a period $T_i$. $K$ depends on the probability of sensing errors (false alarm, $P_{fa}$, for idle periods, and missed detection, $P_{md}$, for busy periods) as well as the number of sensing events within a period, which in turn depends on the real period duration $T_i$ and the sensing periodicity $T_s$. Moreover, the estimated statistics depend not only on the number of sensing errors but also on how the errors are distributed along the period duration $T_i$. In general, $K$ sensing errors result in a single period being observed as $2K+1$ periods ($K$ periods of the opposite type, and $K+1$ periods of the same type). However, this may not be the case when two or more sensing errors are consecutive. From the previous discussion it can be concluded that the number of interrelated factors affecting the estimated statistics, along with their randomness, prevent the analysis of the problem under study by means of analytical approaches without having to make assumptions and simplifications. As a result, a simulation-based approach is adopted in this work.

## III. Simulation method

The study reported in this paper relies on a simulation method based on the following steps (see Figure 1):

1) Generate a sequence of $N$ alternated idle/busy periods, whose durations $T_0/T_1$ are obtained as random numbers drawn from generalised Pareto distributions[1].
2) From the sequence of idle/busy periods obtained in step 1, determine the sequence of idle/busy states ($\mathcal{H}_0/\mathcal{H}_1$) that would be observed in the primary channel when a sensing period $T_s$ is employed and a PSS performance ($P_{fa}=0$ and $P_{md}=0$) is assumed.
3) Based on the sequence $\mathcal{H}_0/\mathcal{H}_1$ obtained in step 2, compute, as depicted in Figure 1(a), the period durations $\widehat{T}_0/\widehat{T}_1$ that would be estimated under a PSS performance ($P_{fa}=0$ and $P_{md}=0$).
4) Introduce random sensing errors in the sequence $\mathcal{H}_0/\mathcal{H}_1$ obtained in step 2. When the channel is idle ($\mathcal{H}_0$), the observed state may randomly change to busy ($\mathcal{H}_1$) with probability $P_{fa}>0$. Similarly, when the channel is busy ($\mathcal{H}_1$), the observed state may randomly change to idle ($\mathcal{H}_0$) with probability $P_{md}>0$.
5) Based on the sequence $\mathcal{H}_0/\mathcal{H}_1$ obtained in step 4, compute, as depicted in Figure 1(b), the period durations $\breve{T}_0/\breve{T}_1$ that would be estimated under an ISS performance ($P_{fa}>0$ and $P_{md}>0$).
6) Determine the impact of ISS performance by comparing the period durations $\widehat{T}_i$ obtained in step 3 (PSS) and the period durations $\breve{T}_i$ obtained in step 5 (ISS) with the original period durations $T_i$ generated in step 1.

Note that the period durations in step 1 can take any positive real value (i.e., $T_i \in \mathbb{R}^+$), while the period durations in steps 3 and 5 are integer multiples of the sensing period (i.e., $\widehat{T}_i, \breve{T}_i = kT_s$, with $k \in \mathbb{N}^+$). Thus, the estimates $\widehat{T}_i$ and $\breve{T}_i$ include an estimation error component with respect to $T_i$ resulting from the employed finite sensing period $T_s$. The error component associated to $T_s$ and its impact on the estimated primary activity statistics is out of the scope of this work. This work focuses on the analysis of the impact of ISS performance. To this end, the estimation errors of both $\widehat{T}_i$ and $\breve{T}_i$ with respect to the real period durations $T_i$ are computed and compared in order to determine the impact of an ISS performance with respect to a PSS scenario.

## IV. Estimation of the minimum

Given a set $\breve{\mathcal{T}}_i = \{\breve{T}_{i,n}\}_{n=1}^N$ of $N$ period durations observed under ISS, the minimum period duration can be estimated as $\breve{\mu}_i = \min_n(\{\breve{T}_{i,n}\}_{n=1}^N)$. The relative error of $\breve{\mu}_i$ with respect to the real minimum $\mu_i$ is shown in Figure 2 as a function of $T_s$, expressed in relative time units (t.u.). This result is valid for any non-zero probability of error and for both idle and busy periods, irrespective of the experienced channel load. The relative error under PSS is also shown for comparison.

As it can be appreciated, the relative error under PSS shows an oscillating pattern, with zeros at the values of $T_s$ that are integer sub-multiples of the real minimum (i.e., $T_s = \mu_i/k$, with $k \in \mathbb{N}^+$). As pointed out in Section III, the estimated durations $\widehat{T}_i$ are integer multiples of the sensing period. Thus, an exact estimation of $\mu_i$ under PSS is possible when $T_s =$

[1] A common assumption widely employed in the existing literature is that idle/busy periods are exponentially distributed. Recent studies, however, have demonstrated that this assumption is unrealistic and that period durations are more accurately described by means of a generalised Pareto distribution [15].

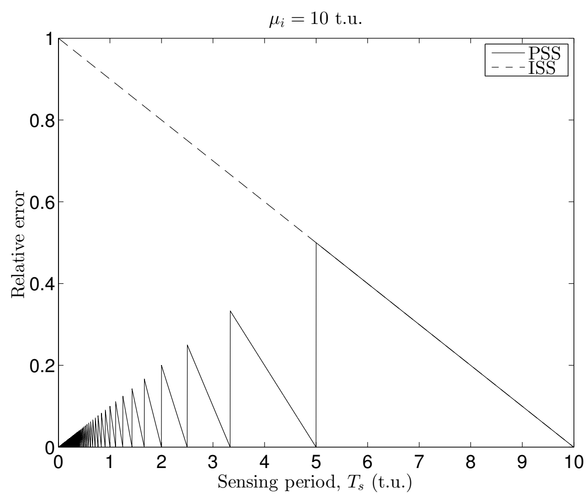


Fig. 2. Relative error of the estimated minimum period $\breve{\mu}_i$.

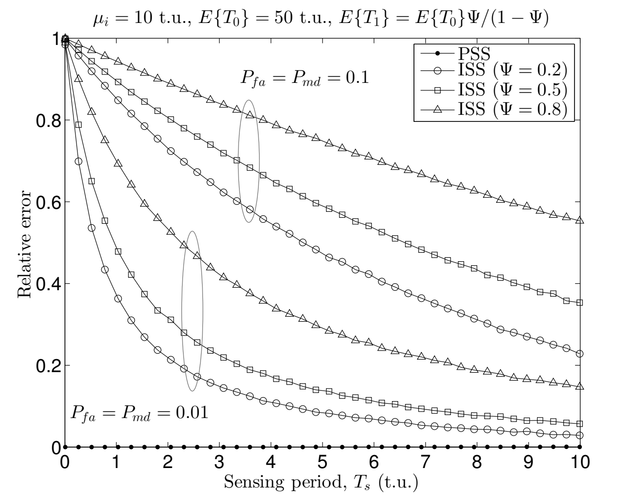


Fig. 3. Relative error of the estimated mean idle period $\mathbb{E}\{\breve{T}_0\}$ as a function of the sensing period $T_s$.

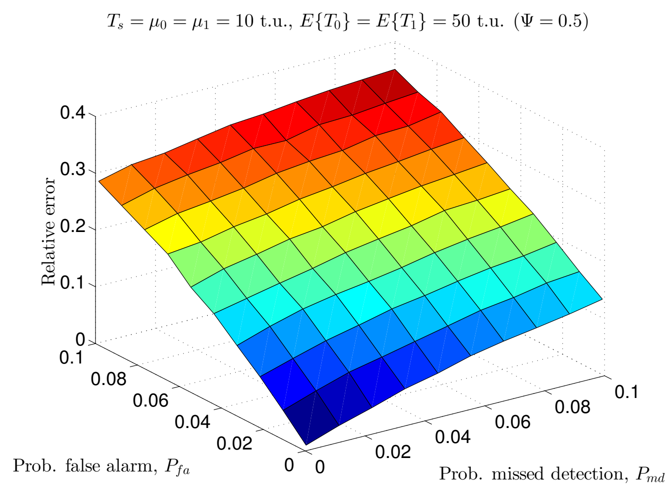


Fig. 4. Relative error of the estimated mean idle period $\mathbb{E}\{\breve{T}_0\}$ as a function of the probabilities of false alarm ($P_{fa}$) and missed detection ($P_{md}$).

$\mu_i/k$. For other sensing periods ($T_s \neq \mu_i/k$) the relative error is greater than zero but in general decreases with $T_s$, meaning that the minimum period $\mu_i$ can be estimated accurately under PSS provided that the selected $T_s$ is sufficiently low.

On the other hand, the opposite trend is observed under ISS (i.e., the relative error increases as $T_s$ decreases), which can be explained as follows. Under ISS, sensing errors can occur at any position within a period. The shortest observable period duration corresponds to the case where a sensing error occurs immediately after or before a channel state transition (from idle to busy or vice versa). In such a case, the channel state is correctly sensed after (before) the transition and the next (previous) sensing event is incorrect so that the time interval elapsed between two observed (but not necessarily real) channel state transitions is equal to the minimum interval between two channel observations (i.e., the sensing period). Therefore, under ISS, $\breve{\mu}_i = T_s$. As a result, the relative error of the estimated minimum under ISS is $|\mu_i - \breve{\mu}_i|/\mu_i = |\mu_i - T_s|/\mu_i$. To guarantee an interference-free operation, the DSA/CR system needs to be able to detect any period of primary activity, which requires $T_s < \mu_i$. For the range of values $T_s \in (0, \mu_i)$, the relative error of $\breve{\mu}_i$ increases when $T_s$ decreases, which explains the behaviour observed in Figure 2. The previous analysis reveals that $\breve{\mu}_i = T_s$ under ISS and it can hence be concluded, as opposed to PSS, that an accurate estimation of the minimum period duration $\mu_i$ of an unknown channel is not possible in the presence of sensing errors, no matter how low the probability of error ($P_{fa}$ or $P_{md}$) is.

## V. Estimation of the moments

The first raw moment (mean) and second central moment (variance) can be used to provide a simple characterisation of a random variable. The estimation errors for the mean and variance of the estimated period durations under ISS were analysed in the context of this work. Since the results obtained for both statistical moments follow similar trends and lead to the same conclusions, only the results for the mean are shown.

Figure 3 shows the relative error of the estimated mean idle period $\mathbb{E}\{\breve{T}_0\}$ under ISS as a function of the sensing period (similar results were obtained for busy periods). As it can be appreciated, the estimation error is higher for shorter sensing periods. When $T_s$ decreases, the number of sensing events within a given period duration increases, and so does the number of potential sensing errors in the period. As discussed in Section II, a higher number $K$ of sensing errors results in the observation of shorter periods, meaning that the difference (i.e., estimation error) between the observed periods $\breve{T}_i$ and the real period duration $T_i$ increases with $K$. Thus, the estimation accuracy of the moments can be improved by reducing the number of sensing events per period (i.e., increasing the employed sensing period). For $T_s < \mu_i$ (see Section IV), the optimum sensing period minimising the estimation error is $T_s \approx \mu_i$, which is corroborated by the results in Figure 3.

The minimum achievable estimation error resulting from $T_s \approx \mu_i$, however, does not necessarily represents an accurate estimation of the moments. As observed in the example of Figure 3, the estimation error may be notably high, even if $T_s \approx \mu_i$, when the probabilities of sensing errors are not sufficiently low. Lowering $P_{fa}$ and $P_{md}$ results in a more accurate estimation of the moments. The relative importance of both parameters on the estimation error is illustrated in Figure 4. It is interesting to note that not only the value of $P_{fa}$ but also the value of $P_{md}$ have an impact on the moments of the estimated idle periods. Since every missed detection itself leads to the observation of an idle period, the value of $P_{md}$ also affects the estimated moments for idle periods. Nevertheless, the moments of the estimated idle periods depend on the value of $P_{fa}$ to a greater extent (similarly, the moments of busy periods are more severely affected by the value of $P_{md}$, but depend on the value of $P_{fa}$ as well). As observed in Figure 4, where $T_s \approx \mu_i$ is assumed, an acceptable estimation of the moments also requires a low probability of sensing errors. Therefore, spectrum sensing methods capable to provide a high detection performance are essential for an accurate estimation of the primary activity statistics from spectrum sensing observations.

It is interesting to note from Figure 3 that the channel load also has an impact on the estimation accuracy. In the simulations, the statistics of idle periods remained unchanged ($\mathbb{E}\{T_0\} = 50$ t.u.) while the statistics of busy periods were adjusted to reproduce different channel loads in terms of the Duty Cycle (DC) $\Psi$ ($\mathbb{E}\{T_1\} = \mathbb{E}\{T_0\}\Psi(1-\Psi)^{-1}$). Higher channel loads (i.e., higher DC values, $\Psi$) are associated to longer busy periods and therefore to a higer number of potential missed detections (i.e., very short idle periods), which leads to a higher estimation error in the moments of idle periods (the same applies for the moments of busy periods as the channel load decreases). Unfortunately, the load/DC of the channel under observation is out of the control of the DSA/CR system. However, as indicated by the results of Figures 3 and 4, by selecting $T_s \approx \mu_i$ and employing spectrum sensing methods with a sufficiently high detection performance, it is possible to estimate the moments of the period durations from spectrum sensing observations within reasonable accuracy levels.

## VI. Estimation of the distribution

Since the original period durations are positive real values ($T_i \in \mathbb{R}^+$), their Cumulative Distribution Function (CDF), denoted as $F_{T_i}(T)$, is continuous. On the other hand, the period durations estimated under PSS and ISS, $\widehat{T}_i$ and $\breve{T}_i$ respectively, are integer multiples of the sensing period ($\widehat{T}_i, \breve{T}_i = kT_s$, $k \in \mathbb{N}^+$), meaning that the CDFs of $\widehat{T}_i$ and $\breve{T}_i$, denoted as $F_{\widehat{T}_i}(T)$ and $F_{\breve{T}_i}(T)$, respectively, are discrete (i.e., $T = kT_s$). Given the impossibility of comparing continuous and discrete distributions, the accuracy of the distribution estimated under ISS is evaluated by considering as a reference the distribution estimated under PSS. A metric widely employed to assess the similarity of two distributions is the Kolmogorov-Smirnov (KS) distance [16], which can be used to compare continuous distributions. For the purposes of this work, the distributions estimated under PSS and ISS can be compared based on the following discretised version of the KS metric [17]:

$$D_{KS} = \sup_k \left| F_{\widehat{T}_i}(kT_s) - F_{\breve{T}_i}(kT_s) \right| \quad (1)$$

Figures 5 and 6 illustrate the estimation error of the distribution for idle periods $F_{\breve{T}_0}(kT_s)$ under ISS, in terms of the KS metric defined in (1), as a function of the sensing period and the probabilities of sensing errors, respectively. The comparison with Figures 3 and 4 indicates that the estimation error of both the statistical moments and the distribution of period durations follow the same dependence with the sensing period $T_s$ and the probabilities of sensing errors ($P_{fa}$ and $P_{md}$). Therefore, the same guidelines provided in Section V for the configuration of spectrum sensing in terms of $T_s$, $P_{fa}$ and $P_{md}$ are also applicable in this case.

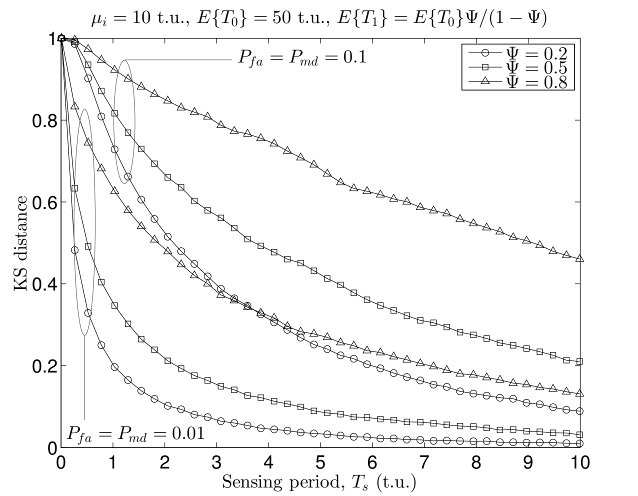


Fig. 5. Estimation error of the distribution for idle periods $F_{\breve{T}_0}(kT_s)$ as a function of the sensing period $T_s$.

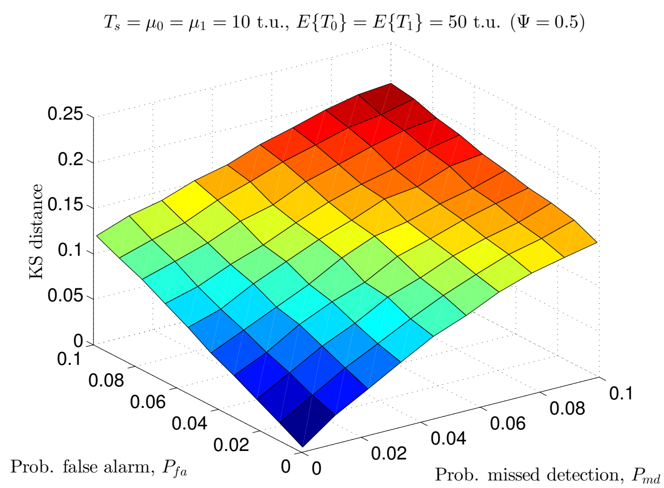


Fig. 6. Estimation error of the distribution for idle periods $F_{\breve{T}_0}(kT_s)$ as a function of the probabilities of false alarm ($P_{fa}$) and missed detection ($P_{md}$).

## VII. Methods to palliate spectrum sensing errors

The presence of errors in the sequence of spectrum sensing decisions leads to incorrectly estimated period durations as illustrated in Figure 1(b). These incorrect values introduce an error component in the estimated primary activity statistics since they are computed based on the whole set of observed period durations. The accuracy of the estimated statistics could be improved if these incorrect values could be identified. In general, it is not possible for a DSA/CR system to determine the reliability of the duration of an activity/inactivity period observed in an unknown primary channel. However, if the real minimum period durations $\mu_i$ of the channel are known, this information can be exploited to detect some incorrect period durations resulting from sensing errors. Note that sensing errors (false alarms and missed detections) in punctual sensing events are observed as periods with a duration $\breve{T}_i = T_s < \mu_i$, which can easily be identified. Moreover, these punctual errors also result in the division of the original period of duration $T_i$ into a number of shorter periods ($\breve{T}_i < T_i$), some of which may be shorter than $\mu_i$ and thus detectable. The knowledge of the real minimum period duration $\mu_i$ can therefore be exploited to palliate the effects of sensing errors. The analysis in Section IV showed that the value of $\mu_i$ cannot be estimated from channel observations in the presence of sensing errors. However, most radio technologies are based on time-slotted frame structures, whose time-slot duration is known and can be considered as the minimum channel (in)activity period (i.e., $\mu_0$ and $\mu_1$).

Given a set $\breve{\mathcal{T}}_i = \{\breve{T}_{i,n}\}_{n=1}^{N}$ of $N$ period durations observed under ISS, the following methods can be employed, when the value of $\mu_i$ is known, in order to palliate the degrading effects of spectrum sensing errors on the estimated statistics:

1) Whenever an observed period has a duration $\breve{T}_{i,n} < \mu_i$, the period $\breve{T}_{i,n}$ is discarded and not used in the computation of the primary activity statistics. This method discards incorrect period durations only.
2) Whenever an observed period has a duration $\breve{T}_{i,n} < \mu_i$, the period $\breve{T}_{i,n}$ is discarded as well as the preceding ($\breve{T}_{i,n-1}$) and subsequent ($\breve{T}_{i,n+1}$) periods since these may presumably be fragments of the original period. This method may discard correct and incorrect periods.
3) Whenever an observed period has a duration $\breve{T}_{i,n} < \mu_i$, an attempt to reconstruct the original period duration is made by summing the period durations $\breve{T}_{i,n-1} + \breve{T}_{i,n} + \breve{T}_{i,n+1}$ and considering the resulting value as a single period duration of the opposite type to $\breve{T}_{i,n}$. This process is combined over all adjacent periods with a duration shorter than $\mu_i$ with the aim of joining all the potential fragments of an original period.

Figures 7 and 8 illustrate the estimation error of the distribution for idle periods $F_{\breve{T}_0}(T)$ under ISS, in terms of the KS metric defined in (1), as a function of the sensing period for medium ($\Psi = 0.5$) and high ($\Psi = 0.8$) channel loads, respectively (similar trends were observed for the relative errors of the estimated statistical moments). As it can be appreciated, the proposed discarding/combining methods are capable to provide significant accuracy improvements with respect to the reference case where all the period duration samples are used as observed in the channel. It is interesting to note that the simplest considered strategy (method 1) provides the best estimation accuracy. Method 2 discards not only incorrect but also correct period durations, which affects negatively the accuracy of the estimated statistics thus leading to a slightly higer estimation error compared to method 1. Method 3 is the least attractive strategy, not only for its higher complexity, but also because it provides the lowest accuracy improvement of the three considered methods, which in some cases is marginal (Figure 8) and in some other cases may even lead to a worse estimation accuracy than the reference case (this was the case observed for $\Psi = 0.2$). The reason is that when several consecutive incorrect period durations (shorter than $\mu_i$) are detected, the observed sequence of idle/busy periods ($\breve{T}_i$) can be the result of different original sequences

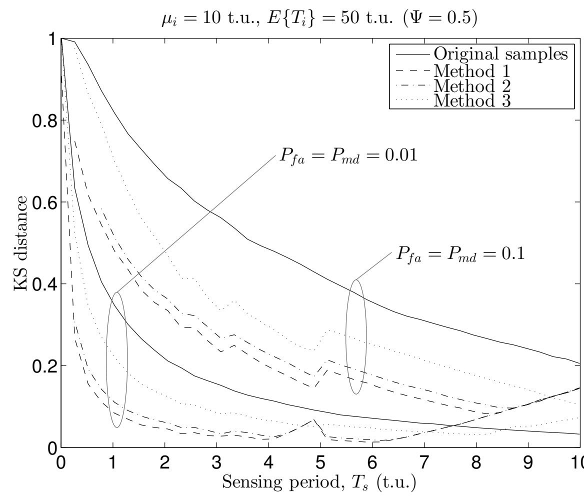


Fig. 7. Estimation error of the distribution of idle periods $F_{\breve{T}_0}(kT_s)$ as a function of the sensing period $T_s$ for medium channel loads ($\Psi = 0.5$).

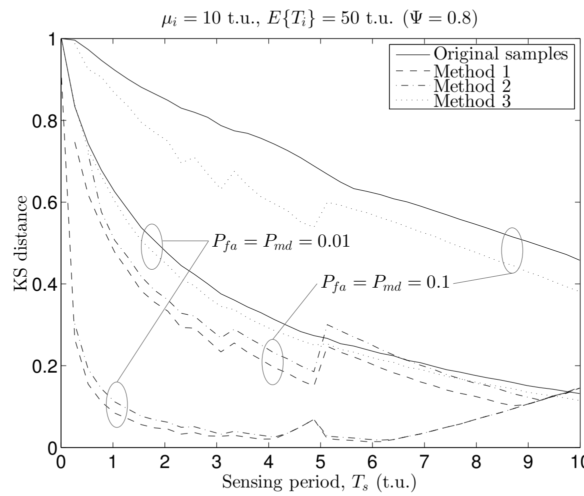


Fig. 8. Estimation error of the distribution of idle periods $F_{\breve{T}_0}(kT_s)$ as a function of the sensing period $T_s$ for high channel loads ($\Psi = 0.8$).

of real periods ($T_i$) depending on whether the errors were caused by false alarms and/or missed detections, which cannot be determined from the observed sequence. As a result, the period durations reconstructed by method 3 may not be the real period durations of the channel and in fact they are not in an appreciable number of cases, which explains the poor accuracy improvement observed for method 3. Based on this analysis, method 1 can be considered the most attractive alternative to palliate the degrading effects of sensing errors on the accuracy of the estimated statistics, not only for its simplicity but also for its attainable accuracy improvements.

The results obtained in Figures 7 and 8 indicate that the first proposed method can provide accurate estimations of the primary activity statistics. For example, the minimum attainable KS distance is $D_{KS} \approx 0.01$ for $P_{fa} = P_{md} = 0.01$ and $D_{KS} \approx 0.1$ for $P_{fa} = P_{md} = 0.1$. However, as opposed to the reference case where all the observed period durations are used in the computation of the statistics, the best attainable accuracy is not obtained for $T_s \approx \mu_i$. In fact, the estimation accuracy for $T_s \approx \mu_i$ is similar to (and in some cases even worse than) the case where no period durations are discarded. When $T_s \approx \mu_i$, periods with a duration equal to the minimum ($T_i = \mu_i$) may be observed as periods with a duration $\breve{T}_i = T_s < \mu_i$. In such a case it is not possible to distinguish between period durations resulting from the real $\mu_i$ and periods incorrectly estimated as a result of false alarms or missed detections. Therefore, method 1 may discard not only incorrect but also correct period durations for $T_s \approx \mu_i$, which explains the trend observed in Figures 7 and 8. When method 1 is employed, the optimum sensing period depends on the particular scenario, but in general an estimation accuracy close to the best attainable accuracy is obtained for $T_s \approx \mu_i/2$ and similar values (experiments performed with other configurations confirmed this statement). This approximation can be used as a simple rule of thumb in the selection of the sensing period. The exact value of the optimum $T_s$ for a particular scenario can be determined by means of simulations based on the method of Section III.

## VIII. Configuration of spectrum sensing

An accurate estimation of the primary activity statistics under ISS conditions is feasible but requires particular considerations to be carefully taken into account. This work has shown how the estimation error depends on three main factors: the channel load ($\Psi$), the employed sensing period ($T_s$) and the probabilities of sensing errors ($P_{fa}$ and $P_{md}$). Only the two latter aspects are under the control of the DSA/CR system, but an appropriate configuration thereof can enable an accurate estimation of the statistics regardless of the channel load.

The optimum sensing period minimising the estimation error is in general $T_s \approx \mu_i$. Since $\mu_i$ cannot be estimated reliably under ISS, the value of $\mu_i$ needs to be known by the DSA/CR system beforehand (in most time-slotted systems, the time-slot duration could be considered as $\mu_i$). In addition to that, accurate estimations require low $P_{fa}/P_{md}$ values, which can be obtained with advanced high-performance sensing algorithms. If the performance of individual DSA/CR sensing nodes does not meet the needed performance levels, cooperative spectrum sensing solutions can be used. In such a case, a central entity can gather individual sensing results to make a final decision, estimate the primary activity statistics and then broadcast this information in the DSA/CR network. Notice that the detection performance of individual DSA/CR sensing nodes can be relaxed by applying discarding methods capable to improve the accuracy of the estimated statistics (method 1), but in this case the optimum value of $T_s$ that minimises the estimation error is not $T_s \approx \mu_i$ and depends on the particular operation scenario, which cannot be determined in a straightforward manner, thus requiring the use of simulations in the design of the optimum sensing period. In some cases, the optimum $T_s$ might be too high to guarantee a required level of spectrum agility in the DSA/CR system. This conflict can be solved by defining an *observation interval* $T_{obs}$ longer than the required sensing period ($T_{obs} = mT_s$, with $m \in \mathbb{N}^+$) and basing the estimation of statistics on such observation interval (i.e., the channel is sensed based on the $T_s$ required to provide a particular spectrum agility, but the the statistics are estimated by taking one channel observation every $m$ sensing events and discarding the observations in the other sensing events).

## IX. Conclusions

DSA/CR systems can benefit from the knowledge of the primary activity statistics, which can be estimated from spectrum sensing decisions. However, the presence of sensing errors resulting from an imperfect sensing performance may result in an inaccurate estimation of the statistics. This work has performed a detailed simulation-based analysis on the accuracy of the estimation of primary activity statistics under imperfect sensing. The relative importance of various parameters has been analysed and several guidelines for the proper configuration of spectrum sensing have been provided. Methods to palliate the degrading effects of spectrum sensing errors on the estimated statistics have been proposed and evaluated as well. The results and conclusions obtained in this work indicate that primary activity statistics can be estimated from spectrum sensing observations within reasonable accuracy levels if appropriate considerations are taken into account.

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