Distribution-Free Spectrum Sensing for Full Duplex Cognitive Radio

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***Abstract*—Recent advancements in Full Duplex (FD) radio have built a promising idea for faster channel sensing in cognitive radio. Full duplex cognitive radio (FDCR) provides an efficient way to utilize the idle channel without interrupting the ongoing transmission. Currently, non-parametric sensing techniques like energy detection and its modified techniques are implemented for FDCR. In this paper, we introduce a Goodness of Fit based distribution-free sensing in FDCR. With Monte Carlo simulations and analytical approximation, we show that the proposed technique outperforms energy detection and other goodness-of-fit based sensing algorithms for FDCR.**

I. Introduction

Cognitive Radio (CR) enables the use of idle Primary User (PU) licensed spectrum by unlicensed users known as Secondary Users (SU). In order to use the licensed spectrum, a SU has to detect if the spectrum can be utilized without causing interference to any PU, which is achieved by means of spectrum sensing [1]. Half-Duplex Cognitive Radio (HDCR) follows a Listen-Before-Talk (LBT) protocol [2] in which the channel needs to be sensed before a transmission can be performed. The Secondary User (SU) opportunistically senses the channel and decides whether it is idle or busy. If the channel is idle, SU can access the channel without causing interference to Primary User (PU). Hence, to avoid this interference, SU must sense the channel frequently while transmitting data. However, in HDCR, the radio has to stop the transmission for sensing which leads to waste of energy and transmission opportunity.

Full Duplex (FD) radio, which enables simultaneous transmission and reception in the same channel [4] overcomes the fundamental limitation of HDCR. To reduce self-interference, recent developments in RF cancellation, analog and digital cancellation have made it possible to develop the FD concept in practice [3]-[6] allowing simultaneous transmission and reception with one and two antennas. Full-Duplex Cognitive Radio (FDCR) makes use of Listen-And-Talk (LAT) protocol instead of the traditional LBT protocol used by HDCR [2]. Effectively, FDCR enables uninterrupted SU transmissions.

The use of Energy Detection (ED) for spectrum sensing in FDCR has been explored in [7]. The main limitation of ED is the difficulty to distinguish between received signal energy and noise energy in a noisy environment (i.e., under low SNR) which in the context of FDCR becomes more challenging since self-residual signal components act as a noise.

Recently, Goodness of Fit (GoF) based spectrum sensing has become popular for spectrum sensing applications. This approach is derived from GoF based hypothesis testing which is used to decide whether the received sample follows a particular statistical distribution. GoF-based sensing compares an empirical distribution of received samples with the distribution of noise under the null hypothesis. Non-parametric GoF testing for spectrum sensing was introduced in [8] with the Anderson-Darling (AD) test. In the same paper, the authors also provided an analytical bound on the performance of statistics. The use of the Kolmogorov-Smirnov (KS) test [9] and the non-parametric sequential detection scheme [10] has also been proposed for spectrum sensing. The application of the Zhang statistic to spectrum sensing was investigated in [11] and analytical expressions for the Zhang statistic based sensing scheme were derived in [12].

Although there exists literature on distribution-free sensing for HDCR, the distribution-free sensing for FDCR remains unexplored. In this paper, GoF based distribution-free sensing for FDCR is introduced. In particular, Likelihood Ratio Statistic G^2 (LRS-G^2) is used to perform GoF testing [13]. Assuming perfect knowledge of the noise power, the distribution of received samples can be compared with the distribution of the sum of noise and the residual signal. Hence, when the empirical distribution closely follows the expected distribution of null hypothesis, it is more likely that the received samples do not contain any PU signal and it can be decided that channel is idle. Moreover, because of the higher statistical power of the Likelihood Ratio Statistic (LRS) test, the proposed technique provides better detection performance. The proposed scheme also allows a reduction on the number of samples for the same detection probability, effectively decreasing the sensing time.

The major contributions of this work are summarized below:

- A distribution-free GoF based LRS-G^2 technique for spectrum sensing in FDCR is proposed and the performance is analyzed under AWGN channel based on the complementary Receiver Operating Characteristic (ROC) for different values of received SNR at the PU node. The impact of various PU/SU SNR is also investigated.
- Given that the distribution of sampled values of Zhang statistic can not be analytically expressed, a large set of samples of the statistic is created using Monte Carlo simulations and distribution-fitting is then used to estimate the distribution which gives the analytical expression of the probability of missed detection.
- The proposed scheme is also analyzed under a more realistic non-Gaussian noise environment (based on the

Middleton's Class A interference model [14]) to consider the effect of impulsive noise. Note that although the AWGN model accurately models the white gaussian noise, it does not model the behavior of commonly occurring interference signals including impulsive noise.

The rest of the paper is structured as follows. Section II presents the system model and assumptions for performance analysis of the proposed scheme. The proposed technique is described in Section III along with its mathematical analysis in Section IV. The performance of the proposed scheme is assessed and compared with energy detection in Section V. Finally, Section VI summarizes and concludes the paper.

II. System Model

Let $\mathbf{x} = [x_0\ x_1\ ...\ x_{N-1}]^T$ be N samples of the transmitted PU signal and $\mathbf{d} = [d_0\ d_1\ ...\ d_{N-1}]^T$ be N samples of the signal transmitted by SU. The corresponding N samples of the received signal, $\mathbf{y} = [y_0\ y_1\ ...\ y_{N-1}]^T$, can be expressed as

$$\mathbf{y} = \sqrt{\rho_p}\, h_p\, \mathbf{x} + \sqrt{\rho_s}\, h_s\, \mathbf{d} + \mathbf{n} \tag{1}$$

where ρ_p is the Signal-to-Noise ratio (SNR) of the PU signal at the SU sensing antenna, h_p is the channel coefficient between the PU and the SU sensing antenna, ρ_s is SNR of the self-residual signal at the SU sensing antenna, h_s is channel coefficient between transmitting and sensing antennas at the receiving node, and $\mathbf{n} = [n_0\ n_1\ ...\ n_{N-1}]^T$ is the noise.

Let x_i and d_i, $i = 0, 1, ...N-1$, be i.i.d. complex gaussian random variables distributed as $\mathscr{CN}(0,1)$. Similarly, for AWGN channel, let n_i be i.i.d. complex gaussian random variable from distribution $\mathscr{CN}(0,1)$.

As already pointed out in Section I, the AWGN channel model can not synthesize a major section of interferences from external and internal sources. Hence, the performance of the proposed scheme will also be analyzed under a non-Gaussian noise model. Under non-Gaussian noise distribution, the CDF of the signal at null hypothesis is different. When Middleton class A noise and Gaussian residual signal are considered, then the resulting distribution under simultaneous transmission and sensing will be the distribution of the summation of a Middleton class A random variable and a complex Gaussian random variable, i.e., the distribution of:

$$Y = \sqrt{\rho_s}\ h_s\ D + W \tag{2}$$

where $D \sim \mathscr{CN}(0,1)$ and the real and imaginary parts of the random variable W, denoted by $\Re(W)$ and $\Im(W)$ respectively, follow the distribution:

$$f_X(x) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m! \sqrt{2\pi\sigma_m^2}} e^{-\frac{x^2}{2\sigma_m^2}} \tag{3}$$

where

$$\sigma_m^2 = \frac{m/A + \Gamma}{1+\Gamma}$$

with A being the impulse index and Γ being the ratio of powers of Gaussian to non-Gaussian components of noise [15].

III. LRS-G^2 Based Sensing for Full Duplex Radio

When the FDCR is inactive (because the PU is still or has recently been active), the problem of spectrum sensing can be formulated as the well-known binary hypothesis testing problem traditionally considered for HDCR where transmission cannot be performed at the same time as sensing. However, when the FDCR is active (i.e., transmitting) then the problem of spectrum sensing is formulated in terms of the following hypothesis testing problem, where the SU signal is also present as the FDCR does not need to stop transmission in order to sense the channel:

$$\begin{array}{lll} H_0: & \mathbf{y} = \sqrt{\rho_s}\ h_s\ \mathbf{d} + \mathbf{n}; & \text{PU is absent} \\ H_1: & \mathbf{y} = \sqrt{\rho_p}\ h_p\ \mathbf{x} + \sqrt{\rho_s}\ h_s\ \mathbf{d} + \mathbf{n}; & \text{PU is present} \end{array}$$

The proposed LRS-G^2 based sensing technique for FDCR compares the distribution of received samples with the distribution of null hypothesis H_0. Hence, if the empirical distribution is close enough to the theoretical distribution at null hypothesis, the test infers that null hypothesis should not be rejected. Hence, PU signal is absent from the channel.

To test the hypotheses using the LRS-G^2 test, a vector $\mathbf{v}$ is defined as the concatenation of real and imaginary parts of the received vector $\mathbf{y}$:

$$\mathbf{v} = [\Re(y_0)\ ...\ \Re(y_{N-1}),\ \Im(y_0)\ ...\ \Im(y_{N-1})]^T \tag{4}$$

where $\Re(\cdot)$ and $\Im(\cdot)$ represent real and imaginary parts, respectively. Note that v_i follows the same distribution as $\Re(v_i)$ and $\Im(v_i)$ because $\Re(v_i)$ and $\Im(v_i)$ are identically distributed. Vector $\mathbf{v}$ is used to test the hypotheses using the LRS-G^2 test under both Gaussian and non-Gaussian noise models.

A. Gaussian Noise

Let $F_0(x)$ be the CDF of signal at null hypothesis. For a given set of received samples, h_s and ρ_s are constants and d_i and n_i are i.i.d. complex random variables distributed as $\mathscr{CN} \sim (0,1)$. Hence, $v_i \sim \mathscr{CN}(0, \sqrt{\frac{\rho_s+1}{2}})$. The CDF of v_i can be defined as follows:

$$F_0(x) = \frac{1}{\sqrt{\pi(\rho_s+1)}} \int_{-\infty}^{x} \exp\left(-\frac{u^2}{\rho_s+1}\right) du \tag{5}$$

As suggested in [11], let the Zhang statistic for GoF based non-parametric sensing be calculated as:

$$Z = \sum_{i=1}^{2N} \left[\log \left\{ \frac{F_0(v_i)^{-1} - 1}{(2N - \frac{1}{2})/(i - \frac{3}{4}) - 1} \right\} \right]^2 \tag{6}$$

Decisions are made by comparing Z with a decision threshold λ, which is typically calculated so as to meet a required false alarm probability P_f, defined as $P_f = \Pr(Z > \lambda | H_0)$. If $Z > \lambda$, then the null hypothesis must be rejected and hence the PU signal is assumed to be present. On the other hand, if $Z \leq \lambda$, then the null hypothesis must not be rejected and the PU signal can be assumed to be absent from the channel.

B. Non-Gaussian Noise

To analyze the performance in environments where the behaviour of interference cannot be modeled as AWGN, we consider the Middleton's class A noise model in (3).

Let $F_0(x)$ be the CDF of a sample v_i in (4). The distribution is given by $f_0(x) = f_X(x) * f_W(x)$ [16] for $X \sim \mathscr{CN}(0, \rho_s)$:

$$f_0(x) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left[\mathscr{N}(0, \sigma_m^2) * \mathscr{N}(0, \rho_s/2) \right] \quad (7)$$

$$= e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left[\frac{1}{\sqrt{\pi(2\sigma_m^2 + \rho_s)}} e^{-\frac{x^2}{2\sigma_m^2+\rho_s}} \right] \quad (8)$$

Let $F_N(v)$ be the empirical CDF of the elements of $\mathbf{v}$:

$$F_N(v) = \frac{|\{v_i : v_i \leq v, i = 0, 1, ...N-1\}|}{N}$$

The hypothesis testing problem can then be re-written as:

$$\begin{array}{lll} H_0: & F_{2N}(v) = F_0(v); & \text{PU is absent} \\ H_1: & \text{otherwise}; & \text{PU is present} \end{array}$$

Note that $F_{2N}(v)$ is here used because the length of $\mathbf{v}$ is $2N$ when the received sample size is N.

The new statistic for GoF based non-parametric sensing is:

$$Z = \sum_{i=1}^{2N} \left[\log \left\{ \frac{F_{2N}(v_i)^{-1} - 1}{(2N - \frac{1}{2})/(i - \frac{3}{4}) - 1} \right\} \right]^2 \quad (9)$$

which is compared to the threshold λ to select an hypothesis.

IV. Analytical Performance

For a fixed false alarm probability P_f, the performance of spectrum sensing can be assessed in terms of the probability of detection of a signal (P_d), which for the proposed method can be expressed as $P_d = \Pr(Z > \lambda | H_1)$. In this section, we estimate the analytical expression of P_d for the proposed scheme.

As mentioned in [13], the distribution of the Zhang statistic Z can not be determined analytically for the whole range of parameters. In this work, a model for the distribution of the statistic Z is derived by means of Monte Carlo simulations (with 100,000 iterations) where a large sample set of the statistic Z was generated under hypothesis H_1 (considering an appropriate range of parameters) and then fitted to various distribution models. The analysis carried out indicated that the statistic Z can be modeled as a log-normal distribution:

$$f_Z(z) = \frac{1}{z\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln z - \mu)^2}{2\sigma^2} \right)$$

The parameters of the log-normal distribution vary with ρ_p, ρ_s and N. The parameters and corresponding negative log likelihood ratio of the log-normal distribution for various values of the PU signal SNR (ρ_p), with $N = 5$ and $\rho_s = 6dB$, are shown in Table I (simultaneous sensing and transmission) and Table II (sensing-only). The negative log likelihood ratio shown in table is averaged over number of samples. Once the parameters are estimated for the given ρ_p, ρ_s and N using distribution fitting, threshold can be estimated for a given P_f.

From the tables it can be observed that with the increase in ρ_p the mean of the distribution increases as well. Hence, with the increase in ρ_p, the distance between the distribution at null hypothesis and the distribution at the alternate hypothesis increases, thus the decision becomes accurate.

TABLE I: Distribution parameters for log-normal distribution for simultaneous transmission and sensing ($N = 5$, $\rho_s = 6\ dB$).

ρ_p (dB)	μ	σ	NLogL/10^5
-5	1.9334±0.0044	0.7412±0.0032	3.0528
1	2.1358±0.0050	0.8172±0.0036	3.3529
5	2.5798±0.0060	0.9676±0.0043	3.9658
9	3.5263±0.0072	1.1688±0.0051	5.1006

TABLE II: Distribution parameters for log-normal distribution for sensing-only case ($N = 5$).

ρ_p (dB)	μ	σ	NLogL/10^5
-5	2.1487±0.0098	0.7648±0.0033	3.2995
-3	2.278±0.0051	0.8290±0.0036	3.5102
-1	2.5306±0.0057	0.9179±0.0040	3.8639
1	2.9384±0.0063	1.0164±0.0045	4.3736

The variation in the performance of the proposed scheme with variations in PU SNR ρ_p and SU SNR ρ_s is related to the effective SNR defined as:

$$\rho_{eff} = \frac{\rho_p}{\rho_s + 1} \quad (10)$$

From both the tables it can be observed that the mean values of the distributions are approximately shifted by ρ_s dB. In other words, the value of μ corresponding to ρ_p dB in the simultaneous transmission and sensing case with SU SNR ρ_s dB is approximately the same as μ corresponding to $\rho_p - \rho_s$ dB for the sensing-only case.

Based on the obtained distribution model for Z, an expression for the probability of detection can be obtained as:

$$P_d = \Pr(Z > \lambda | H_1) \quad (11)$$

$$= \int_{\lambda}^{\infty} f_Z(z) dz \quad (12)$$

$$= 1 - \int_0^{\lambda} \frac{1}{z\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln z - \mu)^2}{2\sigma^2} \right] dz \quad (13)$$

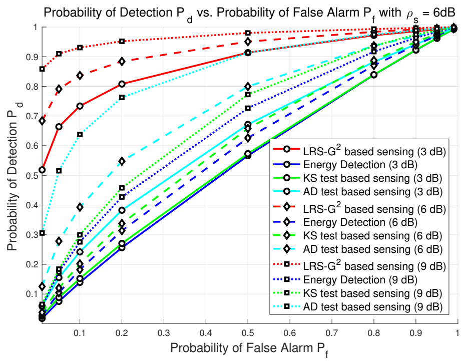
$$= 1 - \left[\frac{1}{2} + \frac{1}{2} \text{erf}\left(\frac{\ln \lambda - \mu}{\sqrt{2}\sigma} \right) \right] \quad (14)$$

$$= \frac{1}{2} - \frac{1}{2} \text{erf}\left[\frac{\ln \lambda - \mu}{\sqrt{2}\sigma} \right] \quad (15)$$

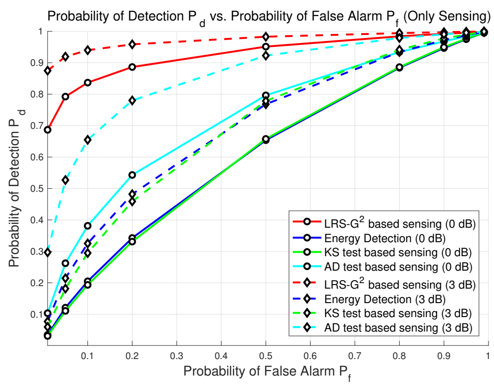
A comparison between simulated and analytical results is provided in the next section.

V. Experiments and Results

In this section, we illustrate the performance of the proposed LRS-G^2 based sensing technique with Monte Carlo simulations. We consider a system with a PU transceiver and a SU FDCR transceiver. We focus on the results for the simultaneous transmission and sensing case as well as sensing-only case. Note that the sensing-only case is essentially same as the conventional HDCR. In general, we consider the number of samples $N = 5$. The number of iterations for Monte Carlo simulations is 100,000. The samples of PU signal and self-residual signal are complex gaussian i.i.d. random variables. The decision threshold is set to meet the target false alarm probability.



(a) Simultaneous sensing and transmission.



(b) Sensing only.

Fig. 1: ROC curves for ρ_s = 6 dB and $N = 5$.

In the experiments, we compare results with primarily 3 baseline methods: AD test [8], KS test [9], and energy detection. To compare the detection probablity of the proposed scheme with baseline methods, we first show Receiver Operating Characteristic (ROC) curves for the simultaneous transmission and reception case as well as the sensing-only case. Fig. 1 shows the probability of detection (P_d) as a function of the false alarm probability (P_f). For the simultaneous transmission and sensing case, the considered PU SNR values are 3, 6, 9 dB. For the sensing-only case, PU signal SNR values are 0 and 3 dB. We observe that the proposed scheme outperforms energy detection for all values of PU SNR for the given ρ_s and sample size. For example, for the simultaneous transmission and sensing case, for $\rho_p = 3$ dB, $\rho_s = 6$ dB and $P_f = 0.1$, the probabilities of detection for the proposed scheme, KS test based sensing, AD test based sensing and the energy detection are 0.7336, 0.1523, 0.2418 and 0.1391 respectively. Similarly, for the sensing-only case, for $\rho_p = 0$ dB and $P_f = 0.1$, the probabilities of detection for the proposed scheme, KS test, AD test and energy detection based sensing are 0.8368, 0.1933, 0.3812 and 0.2048, respectively.

Considering Fig. 1(a) and Fig. 1(b) together, one can observe that for the sensing-only case the performance is significantly higher than for simultaneous transmission and sensing. Considering the sensing-only case as a special case of the simultaneous transmission and sensing case with $\rho_s = -\infty\ dB$, it can be inferred that the performance of the proposed scheme decreases when ρ_s increases, and increases with ρ_p.

Fig. 2 shows P_{md} as a function of PU SNR ρ_p for different values of SU SNR ρ_s. In this case, values of ρ_s are $-\infty$, -5, and 0 dB ($\rho_s = -\infty$ denotes the sensing-only case). It can be observed that with increment in ρ_p the performance of the proposed technique increases. On the other hand, an increment in ρ_s reduces the performance.

Another interesting observation can be made from Figs. 1 and 2: although FDCR can reduce the sensing time, it should be noted that in the presence of a self-residual signal the performance of sensing scheme degrades. Interestingly, a similar observation was also made in [7].

Fig. 3 shows the accuracy of the expression in (15) obtained to analyse the effect of the probability of false alarm (P_f), PU SNR (ρ_p) and SU SNR (ρ_s) on the probability of missed detection (P_{md}). For the simultaneous transmission and sensing case, the considered PU SNR values are -5, 5 and 9 dB and the SU SNR is 6 dB. For the sensing-only case, the PU signal SNR values are -5, -1, 1 and the SU SNR is 6 dB. As observed, the analytical approximation provides an accurate fit.

Due to certain limitations of the AWGN channel model, we also consider a non-Gaussian noise environment based on the Middleton class A noise model. Fig. 4 shows the probability of detection (P_d) of the proposed scheme under such model (with impulsive index $A = 0.2$ and number of samples $N = 25$) as a function of the ratio of powers of Gaussian components to non-Gaussian components (Γ) for $P_f = 0.05$. For the simultaneous transmission and sensing case, we consider PU SNR values of 2, 6 and 10 dB and SU SNR value of 6 dB. For the sensing-only case, the considered PU SNR (ρ_p) is -5, -2 and 2 dB. From Fig. 4(b) it can be concluded that the performance of the proposed scheme degrades with the value of Γ in the sensing-only case. However, Fig. 4(a) indicates that the performance is almost constant with the value of Γ in the simultaneous transmission and sensing case. This can be explained by considering the self-residual interference as Gaussian noise. Since the self-residual signal power is much greater than the noise power, the ratio of Gaussian to the non-Gaussian part in the total noise does not have a significant effect on the ratio Γ.

VI. Conclusion

This paper has proposed a distribution-free Goodness of Fit-based likelihood ratio statistic (G^2) test for spectrum sensing in FDCR. Monte Carlo simulations and distribution fitting have been employed to obtain an analytical estimation of

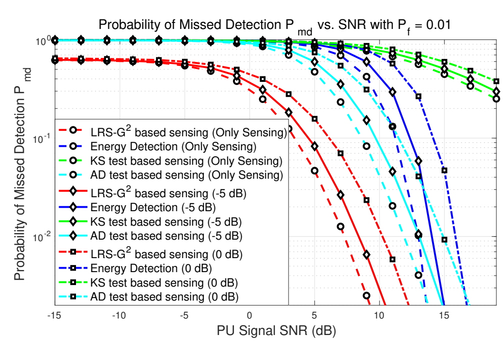
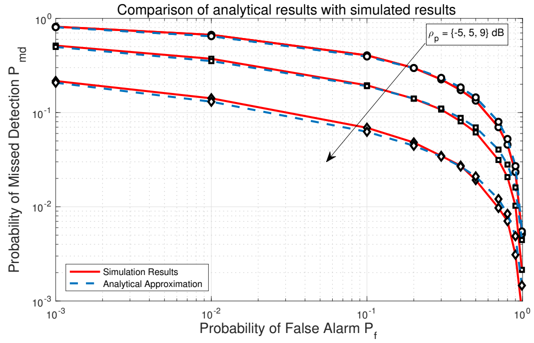
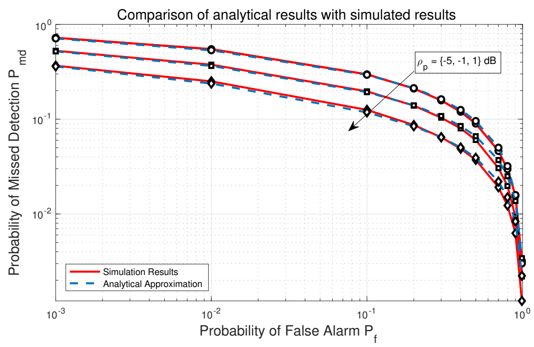


Fig. 2: P_{md} vs. ρ_p for $\rho_s = \{-\infty, -5, 0\}$ dB, $N = 5$, $P_f = 1\%$.

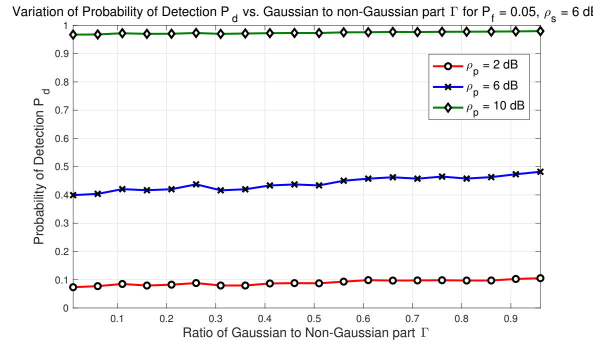


(a) Simultaneous sensing and transmission.

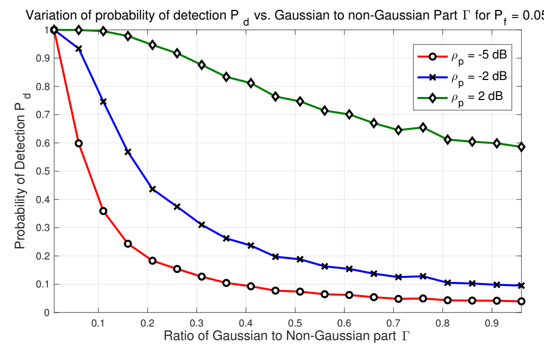


(b) Sensing only.

Fig. 3: Simulation results vs. analytical approximation.



(a) Simultaneous sensing and transmission.



(b) Sensing only.

Fig. 4: P_d vs. Γ of proposed scheme for different values of ρ_p with $P_f = 0.05$ and $\rho_s = 6$ dB.

the performance for the proposed scheme, which is otherwise mathematically untractable. Acknowledging the limits of the AWGN channel model, a non-Gaussian noise environment has also been considered for performance evaluation. The proposed method not only allows CR to perform spectrum sensing without having to stop SU transmissions (which allows a much more efficient exploitation of spectrum opportunities) but also, as demonstrated by the obtained simulation results, outperforms the well-known energy detection method when sensing the PU channel during SU transmissions.

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